

## MATHCOUNTS 2024–2025 Handbook Poster Solution



We can start by choosing the first pair from the 8 friends, which can be done in  $8 \text{ choose } 2 = \frac{8!}{(6! \times 2!)} = \frac{(8 \times 7)}{(2 \times 1)} = \underline{28}$  ways. After selecting the first pair, we are left with 6 friends, and we can choose the second pair in  $6 \text{ choose } 2 = \frac{6!}{(4! \times 2!)} = \frac{(6 \times 5)}{(2 \times 1)} = \underline{15}$  ways. For the third pair, we select 2 out of the remaining 4 friends, which can be done in  $4 \text{ choose } 2 = \frac{4!}{(2! \times 2!)} = \frac{(4 \times 3)}{(2 \times 1)} = \underline{6}$  ways. Finally, the last pair of the remaining 2 friends can only be chosen in  $2 \text{ choose } 2 = \frac{2!}{2!} = \underline{1}$  way. Multiplying these combinations together, we get  $28 \times 15 \times 6 \times 1 = \underline{2520}$  ways. However, since the order in which the pairs are chosen does not matter, we must divide by the number of ways to arrange the 4 pairs, which is  $4! = 24$ . Therefore, the number of distinct ways to pair off the 8 friends into 4 teams of 2 is  $2520/24 = \mathbf{105}$  ways.