

MATHCOUNTS 2023–2024 HB Poster Solution

Hitting a balloon earns Kai **1**, **5** or **10** points. The total score is the **sum of the points** earned by throwing **3 darts**. How many **different total scores** can Kai get?

MATHCOUNTS[®]
www.mathcounts.org
 Get the solution at www.mathcounts.org/poster

Title Sponsors: RTX, U.S. Department of Defense STEM
 Lead Sponsors: BAE Systems, Northrop Grumman Foundation
 National Sponsors: National Society of Professional Engineers, 3M/givex, Texas Instruments Incorporated, Art of Problem Solving
 Founding Sponsors: National Society of Professional Engineers, National Council of Teachers of Mathematics, CNA Insurance

Kai earns 0, 1, 5 or 10 points with each of the 3 darts thrown. We can think of this problem as Kai throwing 3 balls, each of which lands in one of 4 bins. Thinking of it this way allows us to use the *stars and bars* technique to find the total number of outcomes. We will use 3 stars to represent the 3 balls (or darts) and 3 bars as separators for the 4 bins (or point values). The total number of arrangements of 6 items, with 3 of one type (stars) and 3 of another (bars), is $6!/(3! \times 3!) = (6 \times 5 \times 4)/(3 \times 2 \times 1) = 5 \times 4 = 20$ arrangements. The table below shows all 20 outcomes of Kai throwing 3 darts and the total score for each outcome.

		Quantity of Each Point Value																			
Points	0	3	2	2	2	1	1	1	1	1	1	0	0	0	0	0	0	0	0		
	1	0	1	0	0	2	0	0	1	1	0	3	0	0	2	2	1	1	0	0	1
	5	0	0	1	0	0	2	0	1	0	1	0	3	0	1	0	2	0	2	1	1
	10	0	0	0	1	0	0	2	0	1	1	0	0	3	0	1	0	2	1	2	1
Total	0	1	5	10	2	10	20	6	11	15	3	15	30	7	12	11	21	20	25	16	

As shown, 4 of the totals occur twice. Excluding the duplicates, we see that Kai can get $20 - 4 = 16$ different total scores.