# MATHCOUNTS ${ }^{\text {® }}$ Problem of the Week Archive National Bike Month - May 13, 2024 

## Problems \& Solutions

May is National Bike Month, so here are a few problems related to bicycles.
Brenden is riding an old-time bicycle that has a back wheel with a diameter of 1 foot and a front wheel with a radius of 1.5 feet. After traveling 1000 feet, how many more rotations than the front tire has the back tire completed? Express your answer to the nearest whole number.
$1000 \mathrm{ft} /[(0.5)(2)(\pi)]=318.309886$ rotations of the back wheel
$1000 \mathrm{ft} /[(1.5)(2)(\pi)]=106.103295$ rotations of the front wheel
$318.309886-106.103295=\mathbf{2 1 2}$ rotations to the nearest full rotation

Jermaine starts riding his bike at a rate of 10 miles per hour. After 5 minutes, Nelson starts riding from the same starting point as Jermaine and along the same route at a rate of 12 miles per hour. If each boy rides at a constant rate, how many minutes will Jermaine ride his bike for before Nelson catches up with him?

Let's call the time that both boys are riding $t$. We know that once Nelson catches up to Jermaine, the two boys will have each ridden the same distance. Let's call that distance d. Thus, for Jermaine, we can say 10 $=d /(t+5 / 60)$ and for Nelson, we can say $12=d / t$. Let's set both equations equal to $d$.
$d=10(t+5 / 60)$ and $d=12 t \rightarrow 10(t+5 / 60)=12 t$
Now, we can solve for $t$.
$10 t+5 / 6=12 t \rightarrow 5 / 6=2 t \rightarrow t=5 / 12$ hours
We're asked for an answer in minutes, so (5/12)(60) $=25$ minutes. Therefore, Jermaine rode for $25+5=$ 30 minutes.

At Your Town Bike Shop, handle bar streamers are sold in packs of two and beads for spokes are sold in sets of 10. Georgina purchases 2 packs of handle bar streamers and 3 sets of beads for $\$ 27.85$ before tax. Alexis purchases 1 pack of streamers and 4 sets of beads for $\$ 26.30$ before tax. If Nana wants to buy 3 packs of streamers and 6 sets of beads, how much will her bill be before tax?
Let's let h represent the price of one handle bar streamers pack and b represent the price of one set of beads. Since Georgina purchases 2 packs of handle bar streamers and 3 sets of beads for $\$ 27.85$, we can say that $2 h+3 b=27.85$. Since Alexis purchases 1 pack of streamers and 4 sets of beads for $\$ 26.30$, we can also say that $h+4 b=26.30$.

With two equations and two variables, we can solve the system of equations:
$h+4 b=26.30$
$2 h+3 b=27.85$
By solving the top equation for $h$, we can use substitution to solve for $b$.
$h=26.30-4 b$
$2(26.30-4 b)+3 b=27.85$
$52.60-8 b+3 b=27.85$
$5 b=24.75$
$b=4.95$
Substituting this value for b into one of the equations will allow us to solve for $h$ :
$h+4(4.95)=26.30$
$h=26.30-19.80$
$h=6.50$
Thus, 3 packs of streamers and 6 sets of beads cost $3(6.50)+6(4.95)=19.50+29.70=\$ 49.20$.

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