MATHCOUNTS Minis
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Art of Problem Solving
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Warm-Up!

Try these problems before watching the lesson.

1. Point A is on segment $\overline{BC}$ such that $BA : AC = 3 : 2$. If $BC = 45$, then what is the length of $\overline{AC}$?

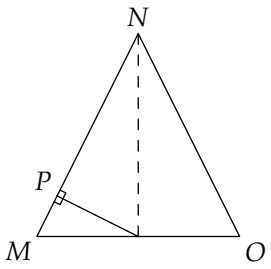
2. An isosceles triangle has legs with length 39 and a base with length 30. What is the area of the triangle?

3. Triangle PQR is a right triangle with $\angle P = 90^\circ$. Point S is on $\overline{QR}$ such that $\overline{PS} \perp \overline{QR}$. If $PS = 6$ and $SR = 8$, then what is PQ? Express your answer as a common fraction.

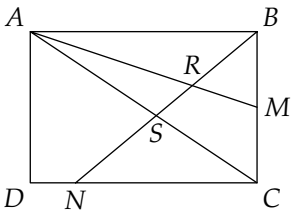
4. Segments $\overline{AB}$ and $\overline{CD}$ are parallel, and segments $\overline{AD}$ and $\overline{BC}$ intersect at point X. If $AB = 14$, $CD = 21$, and $AD = 20$, then what is AX?

The Problem

First Problem: Triangle MNO is an isosceles triangle with $MN = NO = 25$ cm. A line segment, drawn from the midpoint of $\overline{MO}$ perpendicular to $\overline{MN}$, intersects $\overline{MN}$ at point P with $NP : PM = 4 : 1$. What is the length of the altitude drawn from point N to $\overline{MO}$?

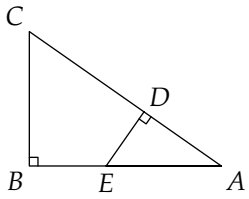


Second Problem: In rectangle $ABCD$, point M is the midpoint of side BC, and point N lies on $\overline{CD}$ such that $DN : NC = 1 : 4$. Segment BN intersects $\overline{AM}$ and $\overline{AC}$ at points R and S, respectively. If $NS : SR : RB = x : y : z$, where x, y, and z are positive integers, what is the minimum possible value of $x + y + z$?

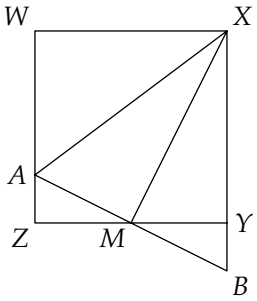


Follow-up Problems

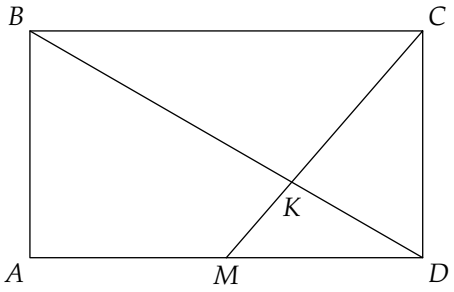
5. In the diagram, angles ABC and ADE are right angles. If $AC = 35$, $AE = 11$, and $BE = 10$, then what is AD? Express your answer as a common fraction.



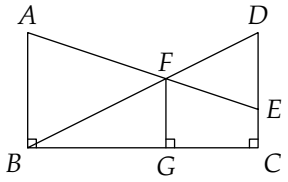
6. In the diagram below, $WXYZ$ is a square, point M is the midpoint of $\overline{YZ}$, and $\overline{AB} \perp \overline{MX}$. If the area of $WXYZ$ is 144 square units, then what is the length of $\overline{AZ}$?



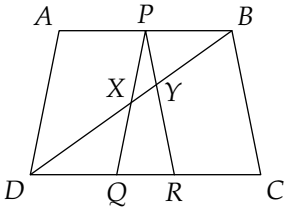
7. In rectangle $ABCD$, $AB = 6$ units, the measure of $\angle DBC$ is 30°, M is the midpoint of segment $\overline{AD}$ and segments $\overline{CM}$ and $\overline{BD}$ intersect at point K. What is the length of segment $\overline{MK}$? Express your answer in simplest radical form.



8. In the diagram below, we have $DE = 2EC$ and $AB = DC = 20$. Find the length of $\overline{FG}$.



9. In the diagram below, $ABCD$ is an isosceles trapezoid with $AD = BC$, and P is the midpoint of $\overline{AB}$. Points Q and R are on $\overline{CD}$ such that $\overline{AD} \parallel \overline{PQ}$ and $\overline{BC} \parallel \overline{PR}$. Diagonal $\overline{BD}$ intersects $\overline{PQ}$ and $\overline{PR}$ at X and Y, respectively. If $CD = 8$ and $AB = 6$, then what is PX/YR? *Source: Mandelbrot Competition*



Wow! Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).