Activity Sheet for the November, 2010, MATHCOUNTS Mini

Warm-Up!

Try these problems before watching the lesson.

1. Find the missing terms in each of the following arithmetic sequences:

   (a) $5, 11, \underline{\quad}, \underline{\quad}, \underline{\quad}$
   (b) $\underline{\quad}, \underline{\quad}, \underline{\quad}, 5, 11$
   (c) $5, \underline{\quad}, \underline{\quad}, \underline{\quad}, 11$

2. Find the missing terms in each of the following geometric sequences:

   (a) $2, 18, \underline{\quad}, \underline{\quad}, \underline{\quad}$
   (b) $\underline{\quad}, \underline{\quad}, \underline{\quad}, 2, 18$

3. The third and fourth terms of an arithmetic sequence are the first second terms of a geometric sequence. If the first two terms of the arithmetic sequence are $5, 2$, then what is the fourth term of the geometric sequence?

4. The first two terms of an arithmetic sequence are 4, 10. What is the first term greater than 1000?

The Problem

First Problem: The fifth term of a geometric sequence of positive numbers is 11 and the eleventh term is 5. What is the eighth term of the sequence?

Second Problem: The positive integers A, B and C form an arithmetic sequence while the integers B, C and D form a geometric sequence. If $\frac{C}{B} = \frac{5}{3}$, what is the smallest possible value of $A + B + C + D$?

Follow-up Problems

5. The fifth term of a geometric sequence of positive numbers is 7 and the twenty-first term is 28. What is the thirteenth term?

6. The third term of a geometric sequence of negative numbers is -2 and the fifteenth term is -162. What is the sixth term of the sequence?

7. Joan is thinking of a geometric sequence with first term 6 and third term 2. John is also thinking of a geometric sequence with first term 6 and third term 2. Must they be thinking of the same sequence?

8. An amoeba is placed in a puddle one day. Every day, the amoeba splits into two amoebas. The puddle is finally completely full of amoebas on the 23rd day. What was the first day on which the puddle was half-full of amoebas?

Further Exploration

The following problems are appropriate for students who have studied quadratic equations.

9. The sequence $1, a, b$ is both a geometric and an arithmetic sequence. Must $a = 1$?

10. Suppose x, y, z is a geometric sequence with common ratio r and $x \neq y$. If $x, 2y, 3z$ is an arithmetic sequence, then what is r? *(Source: AHSME)*

Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).