

This practice plan was created by **Tyler Erb**, a math teacher and coach at Community House Middle School. Tyler created numerous free resources for MATHCOUNTS coaches in his role as the 2021-2022 DoD STEM Ambassador for MATHCOUNTS. Find more resources and information at [dodstem.us](http://dodstem.us).

## Mass Points

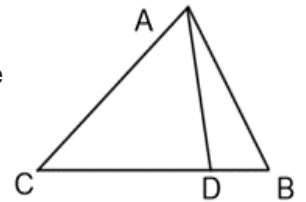


### Warm-Up!

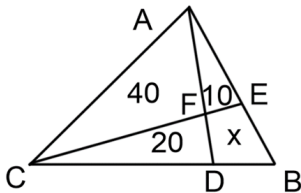
Try these problems before watching the lesson.

- Triangle ABC has medians AE and BF. The point that they intersect is point G. What is the ratio of the length BG to BF? Express your answer as a common fraction.

- CD is 3 times as long as BD. If the area of triangle ABC is  $28 \text{ m}^2$ , what is the area of triangle ABD?



- CD:DB is a 7 to 3 ratio. The area of triangle AFC is  $40 \text{ in}^2$ ; the area of triangle FEA is  $10 \text{ in}^2$ ; and the area of triangle DFC is  $20 \text{ in}^2$ . What is the area of quadrilateral FEBD? Express your answer as a common fraction.



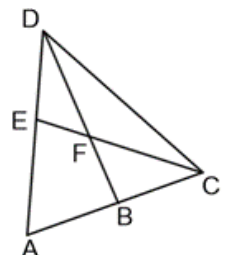
- In triangle ABC, there is a point D on segment AB so that  $AD:DB = 2:5$ . E is on segment CB such that  $CE:EB = 2:3$ . The point where AE and CD intersect is point F. What is the ratio of AF to FE?



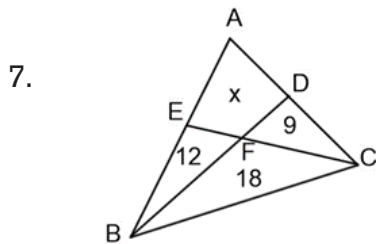
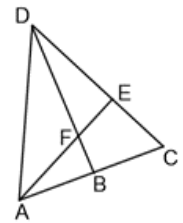
### The Problems

Take a look at the following problems and follow along as they are explained in the video.

- Point E splits segment DA into a 2 to 3 ratio between DE and EA.  $AB:BC = 3:5$ . What is the ratio of DF to FB? Express your answer as a common fraction.

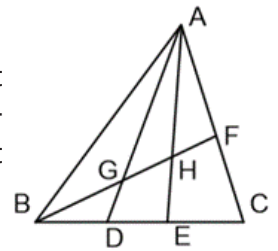


6. Find  $\frac{CE}{DE}$  if  $\frac{DF}{BF} = 5$  and  $\frac{AF}{EF} = 6$ . Express your answer as a common fraction.

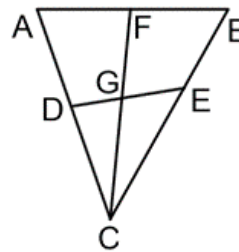


The area of triangle BEF is 12; the area of triangle FBC is 18; and the area of triangle FDC is 9, as shown in the figure. What is the area of quadrilateral EADF? Express your answer as a decimal to the nearest tenth.

8. As shown in the figure, D and E are points on BC of triangle ABC such that  $BD:DE:EC = 1:2:3$ . The median BF meets AD and AE at G and H, respectively, and is divided into lengths  $x$ ,  $y$  and  $z$ . Assuming  $x$ ,  $y$  and  $z$  are the smallest possible integers, find  $x + y + z$ .



9. Point D splits segment AC such that  $\frac{AD}{DC} = \frac{3}{2}$ . F is the midpoint of segment AB.  $\frac{BE}{EC} = \frac{3}{5}$ . What is  $\frac{DG}{GE}$ ? Express your answer as a common fraction.

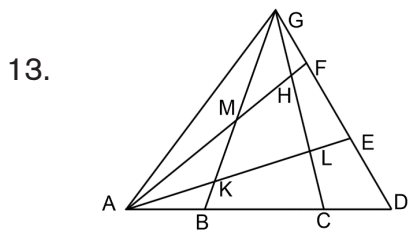
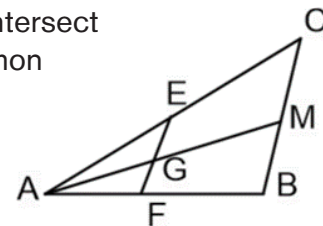


## Piece It Together

Use the skills you practiced in the warm-up and strategies from the video to solve the following problems.

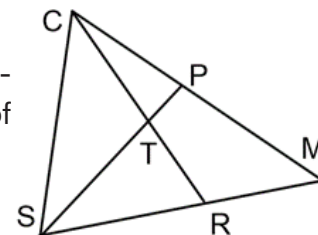
10. In triangle ABC, D is the midpoint of BC. E splits segment AC such that  $AE:EC = 4:7$ . The point where BE and AD intersect is point F. What is the ratio of AF to AD? Express your answer as a common fraction.
11. In triangle GCM, point P lies on segment GC; point R lies on segment CM; and point T lies on segment MG. GP is twice as long as PC, and MR is  $\frac{1}{3}$  the length of RC. The point where median TC and segment PR intersect is point A. What is CA:AT? Express your answer as a common fraction.

12. M is the midpoint of BC, AB = 14, and AC = 16. Segment AM and EF intersect at G. If AE = 3AF, then what is EG/GF? Express your answer as a common fraction.



13. In the diagram,  $AM:MH:HF = 5:3:1$ . We also know  $GM:MK:KB = 6:2:1$ . What is  $FE:ED$ ? Express your answer as a common fraction.

14. Using the diagram, the area of triangle CTS is  $21 \text{ in}^2$ , while the area of triangle STR is  $14 \text{ in}^2$ . The area of triangle CPT is  $12 \text{ in}^2$ . What is the area of quadrilateral PMRT? Express your answer as a common fraction.



## Optional Extension

*To extend your understanding and have a little fun with math, try the following activity.*

You have applied mass points to triangles, but what would it look like in a quadrilateral? In many geometry problems, we may need to add segments to make our drawing easier to interpret or to use properties of figures that we know. As always, we may use mass points here because the problem gives us ratios of side lengths. Can you figure out what to add to the figure to make it solvable?

ABCD is a rectangle with an area of  $96 \text{ in}^2$ . Q lies on AB such that  $AQ:QB = 1:5$ , and P lies on CD such that  $CP:PD = 1:3$ . S is the midpoint of BC. M is the intersection of AS and QP. AB is 12 inches, and AD is 8 inches. Find the area of quadrilateral MSCP. Express your answer as a common fraction.

