MATHCOUNTS Minis
www.mathcounts.org
presented by
Art of Problem Solving
www.artofproblemsolving.com

Activity Sheet for the January, 2010, MATHCOUNTSMini

Warm-Up!

Try these problems before watching the lesson.

1. I have 12 yellow socks and 14 blue socks. If I choose socks at random, what is the smallest number of socks I can choose and still be sure of getting two socks of the same color?

2. I have 14 yellow socks, 9 blue socks, 13 green socks, and 6 black socks. What is the largest number of socks I can possibly choose without choosing two socks of the same color?

3. Each of the integers $1, 2, 3, 4, \ldots, 16$ is written on a separate slip of paper and these slips are placed in a pile. Jillian will randomly draw slips from the pile without replacement and will continue drawing until two of the numbers she has drawn from the pile have the same remainder when divided by 2. What is the maximum number of slips that Jillian can draw without choosing two that have the same remainder when divided by 2? (She stops when any two have the same remainder when divided by 2, not just when two choices in a row have the same remainder.)

4. Each of the integers $1, 2, 3, 4, \ldots, 16$ is written on a separate slip of paper and these slips are placed in a pile. Jillian will randomly draw slips from the pile without replacement and will continue drawing until two of the numbers she has drawn from the pile have different remainders when divided by 2. What is the maximum number of slips that Jillian can draw without choosing two that have different remainders when divided by 2?

The Problem

Each of the integers $1, 2, 3, 4, \ldots, 16$ is written on a separate slip of paper and these slips are placed in a pile. Jillian will randomly draw slips from the pile without replacement and will continue drawing until two of the numbers she has drawn from the pile have a product that is a perfect square. What is the maximum number of slips that Jillian can draw without obtaining a product that is a perfect square? *2009 State Target Round* #8

Follow-up Problems

5. What is the maximum number of slips Jillian can draw if there are 30 slips, numbered from 1 to 30, without obtaining a product that is a perfect square?

6. What is the maximum number of slips Jillian can draw if there are 33 slips, numbered from -16 to 16, without obtaining a product that is a perfect square?

Further Exploration

The following problems can all be solved with the Pigeonhole Principle. See if you can figure out how!

7. Explain why there must be two different non-bald people in New York City who have the same number of hairs on their heads.

8. Show that in any group of 7 integers, two of the integers have the same remainder when divided by 6.

9. I have 46 pieces of candy that I am dividing among myself and 4 of my friends. Explain why at least one of us receives at least 10 pieces of candy.

10. Show that in any group of 8 people, there are two people in the group who have the same number of friends among the people in the group. (This one's pretty tricky!)

Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).