Percentages

Warm-Up!

Try these problems before watching the lesson.

Coach instructions: *Give students around 10 minutes (2 minutes per problem) to go through the warm-up problems.*

1. What number is 10% of 20% of 30% of 40? Express your answer as a decimal to the nearest hundredth.

   To find percent of a number, we convert the percent to a decimal and multiply. Thus, we can express the question as $x = 0.1 \cdot 0.2 \cdot 0.3 \cdot 40$. **Multiplying, we get 0.24.**

2. A restaurant automatically adds an 18% tip to the bill. If the tip was $9, what was the bill before the tip was added, in dollars?

   18% times the bill (*x*) gives the amount of the tip in dollars. 18% as a decimal is 0.18. We can solve the equation $0.18x = 9$ **by dividing both sides by 0.18.** $\frac{9}{0.18} = 50$.

3. A 6% rate increase by a local media cable company resulted in an increase of $1.20 per month on a family's bill. How many dollars was the monthly bill before the increase?

   The initial rate (*x*) times the 6% rate increase is equal to $1.20. We can solve the equation $0.06x = 1.2$ **by dividing both sides by 0.06.** $1.2 / 0.06 = 120 / 6 = 20$.

4. A stock loses 10% of its value on Monday. On Tuesday it loses 20% of the value it had at the end of the day on Monday. What is the overall percent loss in value from the beginning of Monday to the end of Tuesday?

   If its initial value was *x*, after Monday, its value is .9*x*, and after a further 20% loss, its value becomes $0.8 \cdot 0.9x = 0.72x$, **making for a total loss of 28 percent.**

5. The original price of an item was $50. The store deducted 20%, and then deducted an additional 20% off the reduced price. How many dollars more would a consumer save if the store had simply reduced the original price by 40%?

   The price after a 20% deduction is 80% of its original price. After a further 20% deduction, its value becomes $0.8 \cdot 0.8 \cdot 50 = 32$. **A 40% deduction of its original price has a value of** $0.6 \cdot 50 = 30$. **32 − 30 = 2 dollars.**

The Problems

Take a look at the following problems and follow along as they are explained in the video.

6. A toy store manager received a large order of Mr. Slinkums just in time for the holidays. The manager places 20% of them on the shelves, leaving the other 120 Mr. Slinkums in storage. How many Mr. Slinkums were in this order?

   Solution in video. Answer: 150 Mr. Slinkums.

7. Edward is one of the six people who each are writing 180 math problems. When he solves every problem, he gets an incorrect answer for 10% of the problems that he wrote and for 5% of the problems written by the others. For what fraction of the problems does Edward get the wrong answer? Express your answer as a common fraction.

   Solution in video. Answer: 7/120.

8. The length of a rectangle is twice its width. If the length is decreased by 20% and the width is increased by 20%, by what percent is the area decreased?

   Solution in video. Answer: 4 percent.

Piece It Together

Use the skills you practiced in the warm-up and strategies from the video to solve the following problems.

9. After deducting his 10% commission, Jun sent $27 to the newspaper dealer for whom he delivers papers. If each newspaper sells for 20 cents, how many papers did Jun deliver?

   The $27 that Jun sent back to the dealer was 90% of his revenue, meaning that he brought in a total of 27 ÷ 0.9 = 30 dollars. For Jun to collect that much money, he must have delivered $\frac{30}{0.20}$ **= 150 newspapers.**

10. During the first year, ABC's stock price starts at $100 and increases 100%. During the second year, its stock price goes down 25% from its price at the end of the first year. What is the price of the stock, in dollars, at the end of the second year?

    Over the first year, a 100% increase of $100 would make the stock price $200. A 25% decrease would leave the stock value at 75% of the $200. 0.75 · 200 = 150.

11. Otto's investment portfolio consisted of shares of internet stock and copper stock. During the year, the value of his internet shares increased 10%, but the value of his copper shares decreased from $10,000 to $9,000. During the same year, the total value of his portfolio increased by 6%. What was the dollar value of his internet shares at the end of the same year?

Let x be the initial value of Otto's internet stock, in dollars. At the beginning of the year, the total value of his portfolio is x + 10,000 dollars. At the end of the year, the total value of his portfolio is 1.1x + 9,000 dollars. Since increasing by 6% is equivalent to multiplying by 1.06, we have $1.06(x+10,000)=1.1x+9,000$. **Distributing and collecting terms, we find x = 40,000. At the end of the year, his internet stock is worth 40,000 · 1.1 = 44,000 dollars.**

12. A consumer report revealed the following information about three tubes of toothpaste. Bright is 60% more expensive than Fresh and has 25% less volume than Glow. Glow is 25% less expensive than Bright and has 33.$\overline{3}$% more volume than Fresh. Fresh costs $1.00 per unit of volume. What is the number of cents per unit of volume of Glow?

Make a table to organize the information about the cost and the volume of the three tubes of toothpaste. Let the number of units of volume in one tube of Fresh be u. Then the cost of one tube of Fresh is \$$u$. Since Bright is more expensive than Fresh by 60%, one tube of Bright costs \$ $\frac{8}{5}u$. Also, since Glow has $33\frac{1}{3}$% more volume than Fresh, the volume of a tube of Glow is $\frac{4}{3}u$ units.

	volume	cost
Bright		$\$\frac{8}{5}u$
Fresh	u	$\$u$
Glow	$\frac{4}{3}u$	

Finally, multiply the volume of Glow by ¾ to find the volume of Bright, and multiply the cost of Bright by ¾ to find the cost of Glow.

	volume	cost
Bright	u	$\$\frac{8}{5}u$
Fresh	u	$\$u$
Glow	$\frac{4}{3}u$	$\$\frac{6}{5}u$

Dividing the cost of a tube of Glow by its volume, we find a cost-per-unit-of-volume of

$$\frac{(\frac{6}{5})u}{(\frac{4}{3})u}=\frac{6}{5}\cdot\frac{3}{4}=\frac{9}{10}= \text{90 cents.}$$

Optional Extension

To extend your understanding and have a little fun with math, try the following activities.

Consider each of the following shopping scenarios and make a decision for each. Be prepared to defend your answer with math!

a. Would you rather use a 70% discount coupon or a 40% discount, 20% discount, then a 10% discount coupon? (Or are both options going to provide an equal discount?)

    A 70% discount will give a final cost of 100% – 70%, or 30% of the original cost, 0.3*x*. A 40% discount would give a cost of 0.6*x*. A 20% discount off of that would leave a cost of $0.8(0.6x)$ and a final 10% discount would give a final cost of $0.9(0.8(0.6x)) = 0.432x$. The first coupon will require you to only pay 30% of the original cost, which is better than paying 43.2% of the cost in all cases.

b. Would you rather use a coupon worth $20 off your entire purchase or 20% off your entire purchase? (Or are both options going to provide an equal discount?)

    A $20 coupon is better for costs under $100. A 20% off coupon is better for totals over $100. At $100, both coupons provide the same discount.

c. Would you rather use a 5% discount coupon but have to pay 5% shipping for the item purchased, or just pay the flat cost of the item with no discount but a free shipping promotion?

    A 5% discount gives a cost of $0.95x$. 5% shipping is an additional cost, so totals $1.05(0.95x) = 0.9975x$. The final item costs 99.75% of what it should have originally, which is a small discount compared to the second option, which is simply 100% of the cost.

d. Come up with your own 'would you rather' scenario that someone might not predict or expect to be the better mathematical option.