

MATHCOUNTS[®]

Divisibility Rules



Warm-Up!

Try these problems before watching the lesson.

1. Two single-digit positive integers have a product of 72. What is their sum?
2. What is the sum of all two-digit multiples of three that have units digit 1?
3. What is the remainder when 38 dozen is divided by 7?
4. What is the least common multiple of 6, 10 and 14?
5. Kris multiplies the first six positive prime numbers together. How many zeros follow the last non-zero digit of the product?



The Problems

Take a look at the following problems and follow along as they are explained in the video.

6. Using each of the four digits 2, 4, 6 and 8 exactly once, how many four-digit multiples of 4 can be formed?
7. How many zeros are there after the last nonzero digit of $125!$?
8. What is the greatest number that evenly divides the sum of any six consecutive whole numbers?



Piece It Together

Use the skills you practiced in the warm-up and strategies from the video to solve the following problems.

9. Ayasha, Beshkno, and Chenoa were all born after 2000. Each of them was born in a year after 2000 that is divisible by exactly one of the prime numbers 2, 3 or 5. Each of these primes is a divisor of one of the birth years. What is the least possible sum of their birth years?

10. The product of three consecutive integers is 157,410. What is their sum?

11. What is the least positive integer n for which $165 \times 513 + 10n$ is a multiple of 9?

12. What is the least positive integer greater than 20 that has exactly three positive factors?

13. What is the least positive integer n such that the value of $2014!/n!$ does not have a units digit of zero?

14. How many positive 3-digit integers are palindromes and multiples of 11?

15. The six-digit number 357,abc has six distinct digits and is divisible by each of 3, 5 and 7. What is the smallest possible value of $a + b + c$?



Optional Extension

To extend your understanding and have a little fun with math, try the following activities.

So you have learned and applied some divisibility rules for various integers, but why do these rules work? Work your way through the divisibility rules and try to explain or prove them. Some might be easier for you than others. Start with 2 and work your way up or bounce around to the ones that make the most sense to you. Try to explain as many as you can!

Divisibility Rules	
#	Rule
2	The number has a units digit 0, 2, 4, 6 or 8.
3	The sum of the digits is a multiple of 3.
4	The number formed by the last two digits is divisible by 4.
5	The number has a units digit 0 or 5.
6	The number is even, and its digits sum to a multiple of 3.
7	The result of subtracting twice the units digit from the number formed by the remaining digits is divisible by 7.
8	The number formed by the last three digits is divisible by 8.
9	The sum of the digits is a multiple of 9.
10	The number has a units digit of 0.
11	The alternating addition and subtraction of the digits is a number divisible by 11.