MATHCOUNTS Minis
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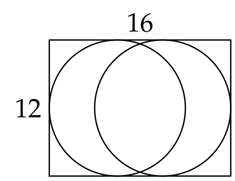
Activity Sheet for the March, 2013, MATHCOUNTS Mini

Warm-Up!

Try these problems before watching the lesson.

1. In the diagram below, the rectangle has length 16 and width 12. Each circle is tangent to three sides of the rectangle. What is the distance between the centers of the circles?

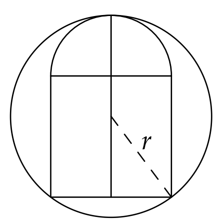


2. Point A is on circle $\mathcal{C}$ and point P is outside the circle such that $AP = 12$ and $\overline{AP}$ is tangent to the circle. If the circle has area 256π square units, then how far is P from the center of the circle?

3. A circle is tangent to all three sides of $\triangle ABC$. The circle meets $\overline{AB}$ at point F and $\overline{AC}$ at point E. If $AB = 14$, $AC = 15$, and $AE = 6$, then what is the length of $\overline{BC}$?

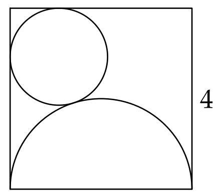
The Problem

First Problem: A silo-shaped figure is formed by positioning a semicircle above a square. The diameter of the semicircle is 2 units long and coincides with the top of the square. What is the radius, r, of the smallest circle that contains this figure?



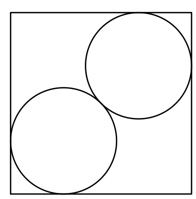
Second Problem: A right triangle has sides with lengths 8 cm, 15 cm, and 17 cm. A circle is inscribed in the triangle. In centimeters, what is the radius of the circle?

Third Problem: A semicircle and a circle are placed inside a square with sides of length 4 cm, as shown. The circle is tangent to two adjacent sides of the square and to the semicircle. The diameter of the semicircle is a side of the square. In centimeters, what is the radius of the circle?



Follow-up Problems

4. In the diagram below, two congruent circles are tangent to each other, and each circle is tangent to two sides of the square. If the side length of the square is 4 units, then what is the radius of each circle? Express your answer in simplest radical form.



5. Two circles with radii 16 and 9 are tangent to each other, and are tangent to line ℓ at distinct points P and Q. Find the length of $\overline{PQ}$.

6. A circle is inscribed in a right triangle with legs of lengths a and b, and hypotenuse of length c. Use the process described in the video to find a formula for the radius of the circle in terms of a, b, and c.

7. Find Harvey's area-based solution to the Second Problem in the video. Then, use that process to find a different formula for the radius of the circle described in Question 6. Your answer should be a formula in terms of a, b, and c. Why do these two formulas give the same value for the radius?

Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).