Activity Sheet for the October, 2012, MATHCOUNTS Mini

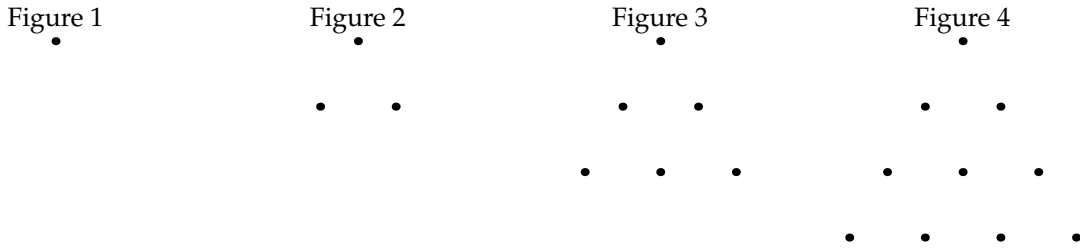
Warm-Up!

Try these problems before watching the lesson.

1. Find the seventh term in the geometric sequence

$$2, 6, 18, \ldots .$$

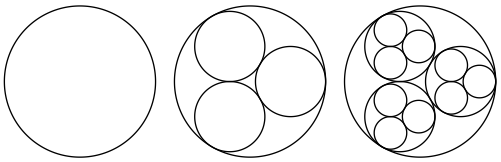
2. A series of figures are created with dots as shown below. After Figure 1, each figure is created by adding a new row of dots that has one more dot than the previously added row of dots. How many dots total are there in Figure 10?



3. If $S(n)$ is a function that returns the sum of the first n positive integers, then what is $S(20) - S(19)$?

The Problem

First Problem: The figure shows the first three stages of a fractal. If the pattern continues, how many circles will Stage 5 of the fractal contain?



Second Problem: The sum of the first n terms of a sequence, $a_1 + a_2 + \cdots + a_n$, is given by the formula $S_n = n^2 + 4n + 8$. The sum of the first three terms, for example, is $S_3 = (3^2)^2 + 4(3) + 8 = 29$. What is the value of a_6?

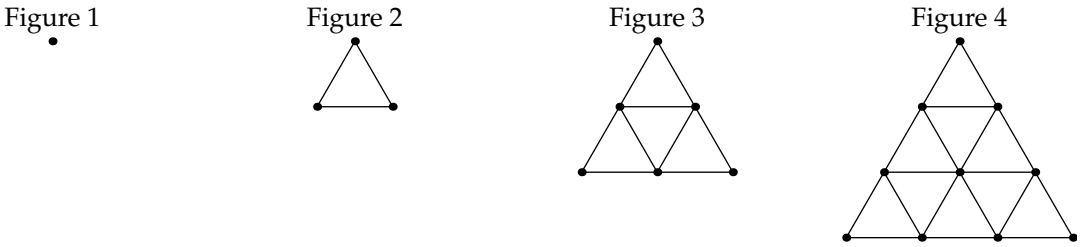
Follow-up Problems

4. What is the sum of the first 9 terms of the geometric sequence

$$3, -6, 12, -24, \ldots?$$

5. Jenny has a list of twelve numbers. For each integer n from 1 to 12, the product of the first n numbers in the list is n^2. What is the last number in the list?

6. We draw the same sequence of Figures as in Problem 2, but we connect the dots in each Figure following the pattern shown below. If each segment from a dot to its nearest neighbors has length 1, then what is the total length of all of the segments in Figure 10?



7. Billy has a lot of time on his hands, so he constructs an infinite sequence of numbers. In Billy's sequence, the n^{th} term is $\frac{n}{3^n}$, so his sequence starts

$$\frac{1}{3}, \frac{2}{9}, \frac{3}{27}, \frac{4}{81}, \ldots.$$

What is the sum of all of the terms in Billy's sequence?

Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).