MATHCOUNTS Minis September 2011 Activity Solutions

Warm-Up!

1. If the result is 35 when you double the number and add seven, we will start by subtracting 7 from 35 to get 28. So, our number is half of 28, which is **14**. To solve algebraically, we first let n represent our unknown. Then we have $2n + 7 = 35$. Subtracting 7 from each side yields $2n = 28$. Then we divide each side by 2 to get $n =$ **14**.

2. We are told that $x = y + 3$ and $y = z - 5$, which can be rewritten as $y + 5 = z$. We are asked to determine the value of $z - x$. Substituting we get $(y + 5) - (y + 3) = y + 5 - y - 3 = 5 - 3 =$ **2**.

3. In the given expression, $x\left(\frac{2y+3}{x} - \frac{2y}{x}\right)$, notice that the expression in parentheses is being multiplied by x, which is the denominator of the two fractions. The xs cancel, $\cancel{x}\left(\frac{2y+3}{\cancel{x}} - \frac{2y}{\cancel{x}}\right)$, and the result is $2y + 3 - 2y =$ **3**.

4. We are told that the television costs \$299 and the older sibling will pay \$45 more than the younger sibling. That means that the other $299 - 45 = 254$ dollars will be split equally between the two siblings. Therefore, the younger sibling will pay $254 \div 2 =$ **127** dollars.

5. From the information given, we can write the following two equations, where x represents the weight of Tweedledee and y is the weight of Tweedledum: $x + 2y = 361$ and $2x + y = 362$. Adding the two equations we get $3x + 3y = 723$. Dividing each side by 3 we see that the sum of their weights is $x + y =$ **241** pounds.

The Problem is solved in the MATHCOUNTS Mini.

Follow-up Problems

6. If we let n represent the numerator and d represent the denominator then the fraction of which Terry is thinking can be written as n/d. From the second sentence we see that $(n + 3)/d = 1 \rightarrow d = n + 3 \rightarrow n = d - 3$. From the third sentence we see that $n/(d - 7) = 2 \rightarrow n = 2(d - 7) \rightarrow n = 2d - 14$. We can set these two expressions equal to each other to get $d - 3 = 2d - 14 \rightarrow d = 11$. That means $n = 11 - 3 = 8$, and Terry's fraction is **8/11**.

7. This problem can be solved several ways. First let's solve it algebraically. We are told that Douglas' favorite number is a positive two-digit integer; let's call it AB where A is the tens digit and B is the units digit. That means that the value of his favorite number is 10A + B. Then a new number is created, AB7, where A now is the hundreds digit, B now is the tens digit and 7 is the units digit. The value of the new number is 100A + 10B + 7. Finally, we are told that the new number is 385 more than Douglas' favorite number. So we have 100A + 10B + 7 = 10A + B +385. Subtracting 10A, B and 7 from both sides yields 90A + 9B = 378. Dividing both sides by 9 gives us 10A + B = **42**. This is Doug's favorite number.

We could also have solved the problem logically by setting up the vertical addition problem:

$$\begin{array}{r} 3\ 8\ 5 \\ +\quad A\ B \\ \hline A\ B\ 7 \end{array}$$

Notice that 5 + B = 7, so B must equal 2. We can then substitute 2 for B in the problem to get:

$$\begin{array}{r} 3\ 8\ 5 \\ +\quad A\ 2 \\ \hline A\ 2\ 7 \end{array}$$

The only integer from 1 to 9 that yields a units digit of 2 when added to 8 is 4. It follows that:

$$\begin{array}{r} 3\ 8\ 5 \\ +\quad 4\ 2 \\ \hline 4\ 2\ 7 \end{array}$$

Thus, Douglas' favorite number is **42**.

8. Let p represent the number of pit bulls, c is the number of chihuahuas and m is the number of mutts. The second sentence of the problem yields the following equations, where A is the total number of dogs: $p = A - 23$, $c = A - 17$, $m = A - 28$ and $A = p + c + m$. If we add the first three equations we get $p + c + m = 3A - 68$. Substituting, we get $A = 3A - 68$. We now solve to determine that the total number of dogs at the pound is $2A = 68 \rightarrow A =$ **34** dogs.

9. If we add the four equations given, the result is $3r + 3y + 3b + 3g = 195$. We can divide both sides of the equation by 3 to get $r + y + b + g = 65$. We can now calculate that there are $r = 65 - 45 =$ **20** red marbles. Likewise, there are $y = 65 - 45 =$ **20** yellow marbles and $b = 65 - 45 =$ **20** blue marbles. There are $g = 65 - 60 =$ **5** green marbles.