



Activity Sheet for the January, 2011, MATHCOUNTS Mini



Try these problems before watching the lesson.

- 1. Without writing anything, find the integer whose square is 13225.
- 2. Without writing anything, find the following:
 - (a) The largest integer whose square is less than one million.
 - (b) The least positive three-digit integer that leaves a remainder of 1 when divided by 7.
 - (c) The least positive four-digit integer that leaves a remainder of 5 when divided by 9.
- 3. Find the largest three-digit integer that leaves a remainder of 3 when divided by 9 and a remainder of 3 when divided by 11.
- 4. Find the largest three-digit integer that leaves a remainder of 7 when divided by 9 and a remainder of 3 when divided by 11.



First Problem: When the square of three times a positive integer is decreased by the integer, the result is 2010. What is the integer?

Second Problem: What is the largest integer less than 2010 that has a remainder of 5 when divided by 7, a remainder of 10 when divided by 11, and a remainder of 10 when divided by 13?



Follow-up Problems

- 5. Seven times the square of an integer plus the integer equals 12390. What is the integer?
- 6. Find the largest integer such that the sum of the integer and its square is less than 8000.
- 7. Find the least positive integer that leaves a remainder of 3 when divided by 5, a remainder of 7 when divided by 9, and a remainder of 7 when divided 13.
- 8. Find the least positive integer that leaves a remainder of 1 when divided by 2, a remainder of 2 when divided by 3, a remainder of 3 when divided by 4, a remainder of 4 when divided by 5, a remainder of 5 when divided by 6, and a remainder of 6 when divided by 7.

Further Exploration

- 9. Johnny picks one of the integers from 0 to 5 and Jane picks one of the integers from 0 to 8. Is it always possible to find a number that leaves Johnny's number as a remainder when divided by 6 and Jane's number as a remainder when divided by 9?
- 10. Johnny picks one of the integers from 0 to 6 and Jane picks one of the integers from 0 to 10. Is it always possible to find a number that leaves Johnny's number as a remainder when divided by 7 and Jane's number as a remainder when divided by 11?
- 11. If you found the same answer for the previous two problems, do the problems again. If you found different answers, then explain why the answers are different. Can you come up with a general rule?

Wow Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).