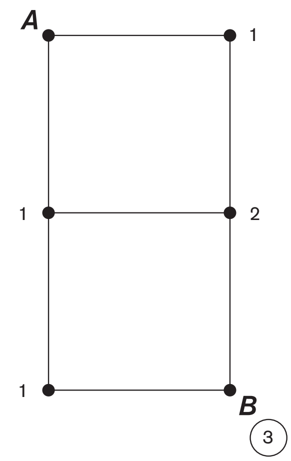
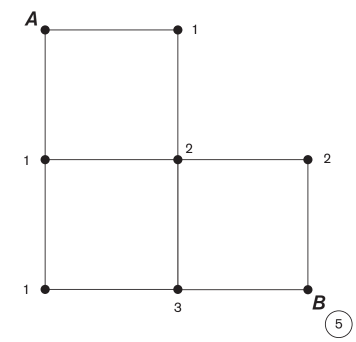
MATHCOUNTS® Minis

March 2010 Activity Solutions

Warm-Up!

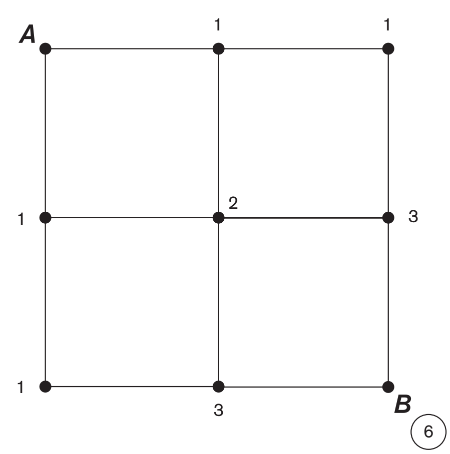


1. There are **3** distinct paths the ladybug could take from point *A* to point *B*: (1) down the two lefthand segments from point *A* and right along the bottom to point *B*, (2) right from point *A* across the top and down the two righthand segments to point *B*, and (3) down one segment on the lefthand side from point *A*, across the center horizontal segment, and down one segment on the righthand side to point *B*. You could also consider the points adjacent to point *B*. Because there is only one way to get from an adjacent point to point *B*, summing the number of paths to the adjacent points to point *B* gives the total number of possible paths to point *B*. As shown in the figure, there is only 1 way to get to the bottom left corner of the figure. There are 2 ways to get to the point above point *B* (the center point on the righthand side of the figure), by going down one segment from point *A* then across one segment or by going across one segment from point *A* then down one segment. Thus, there are 1 + 2 = **3** distinct paths from point *A* to point *B*.



2. Let's consider the adjacent points to point *B*. As shown in the figure, the point to the left of point *B* is where point *B* was located in Problem 1, so in the same way, there are 3 paths to get to this point. Similarly, we find that there are 2 ways to get to the point above point *B*. Thus, there are 3 + 2 = **5** ways to get from point *A* to point *B*.

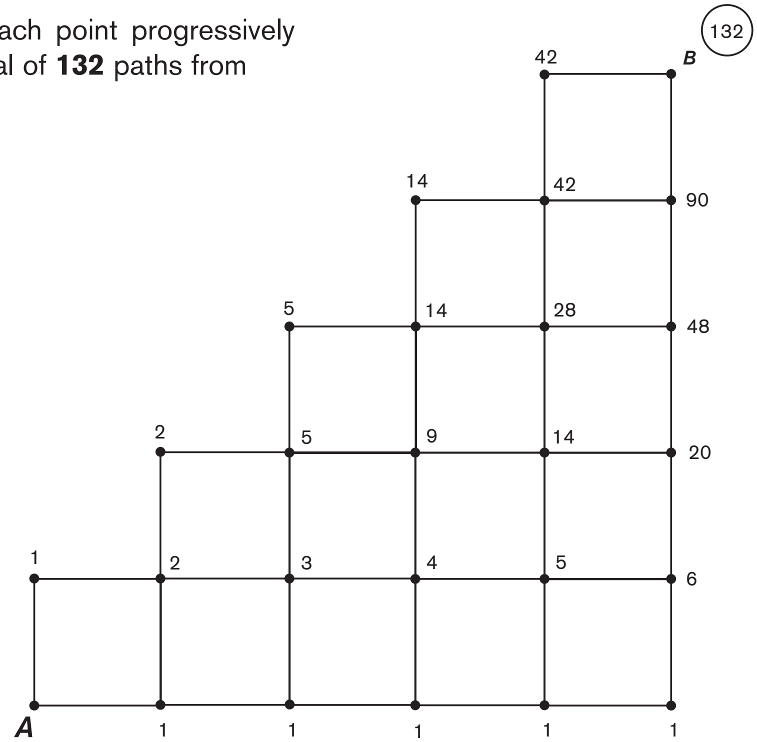
3. This problem continues to build off of Problems 1 and 2. Again, as shown in the figure, the point to the left of point *B* is where point *B* was located in Problem 1, so in the same way, there are 3 paths to get to this point. The additional point in the upper righthand corner of this figure creates 3 paths to get to the point above point *B*: (1) right from point *A* across the two segments at the top, then down one segment; (2) right from point *A* across one segment at the top, down one segment, then right across one segment; and (3) down one segment from point *A*, then right across two segments. So, there are now 3 ways to get to the point above point *B*. Therefore, there are 3 + 3 = **6** paths from point *A* to point *B*.



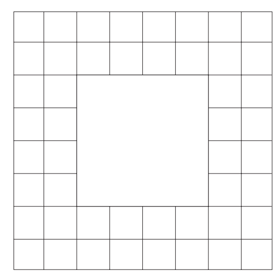
The Problems are solved in the **MATHCOUNTS® Mini** video.

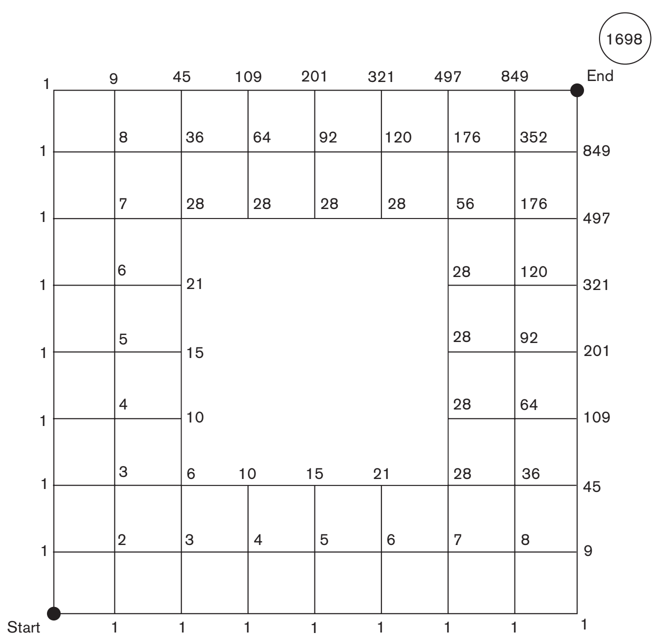
Follow-up Problems

4. Determining the number of paths to each point progressively throughout the figure, as shown, gives a total of **132** paths from point A to point B.



5. From (−4, −4) to (4, 4), considering the outermost 16-step paths creates an 8 by 8 grid of squares, which contains all possible paths within it. Looking at paths that stay outside or on the boundary of the square $-2 \le x \le 2$, $-2 \le y \le 2$ at each step, we're considering the paths in the shape shown at right, from the lower left corner to the upper right corner.

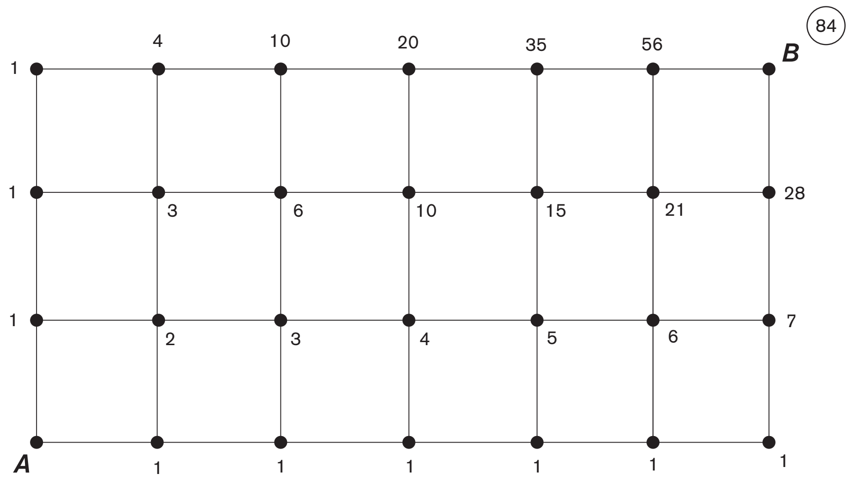




In the figure at left, we can see the number of paths to each point in this figure, giving a total of **1698** paths from (−4, −4) to (4, 4) with the area removed in the middle. All path numbers are filled in here, but you may have also noticed that the number of paths at each point are symmetrical along the line $y = x$ (the line connecting (−4, −4) and (4, 4)), which means half the amount of work to find the answer!

6. There are 6 Rs in the given set of letters, so we can do ${}_9C_6$ to find the number of ways to place the Rs in 9 letter spaces: ${}_9C_6 = 9!/[6!(6 - 3)!] = (9 \times 8 \times 7)/(3 \times 2) = 12 \times 7 = 84$ ways. The Us can then only be placed in the remaining 3 spaces in each scenario, so there are **84** ways to arrange these letters.

7. Determining the number of paths to each point progressively throughout the figure, as shown, gives a total of **84** paths from point *A* to point *B*.



8. In both Problem 6 and Problem 7, we are ultimately performing the operation ${}_9C_6$. In Problem 6, we are selecting 6 spaces from the 9 available letter spaces for the 6 Rs. In Problem 7, we are selecting which 6 steps would be horizontal in the path from point *A* to point *B*.