MATHCOUNTS® Minis December 2009 Activity Solutions

Warm-Up!

1. Five ordered pairs that satisfy this equation are (0, 1), (2, −1), (−1, 1/2), (3, −1/2) and (−1/2, 2/3). (Student answers may differ, as there are an infinite number of possible ordered pairs.)

2. The only ordered pairs of integers that satisfy this equation are (0, 1) and (2, −1).

The Problems are solved in the MATHCOUNTS® Mini video.

Follow-up Problems

3. Multiply the two equations together to get:

$$(x+\frac{1}{y})(y+\frac{1}{z})=1$$

$$xy+\frac{x}{z}+1+\frac{1}{yz}=1$$

Subtracting 1 from both sides of this new equation gives:

$$xy+\frac{x}{z}+\frac{1}{yz}=0$$

Multiply the above equation by z to get:

$$xyz+(x+\frac{1}{y})=0$$

We are given that $x+\frac{1}{y}=2$, so $xyz + 2 = 0$. Therefore, $xyz =$ **−2**.

4. Just as (1/2) × 2 = 1 on the right side of the equation in Problem 3, (1/3) × 3 = 1 as well. Therefore, multiplying these two equations together and following the same process as in Problem 3 will result in the product, $xyz =$ **−3**.

5. As in Problems 3 and 4, (1/4) × 4 = 1, so multiplying these two equations together and following the same process will result in the product, $xyz =$ **−4**.

6. Problems 3, 4 and 5 all produce answers that are the opposite of the given value of $x+\frac{1}{y}$. When multiplying the two provided equations together, as long as the product on the right side of the new equation is 1, then the pattern will continue.

7. Multiply the two equations together to get:

$$(x+\frac{1}{y})(y+\frac{1}{z})=2$$

$$xy+\frac{x}{z}+1+\frac{1}{yz}=2$$

Subtracting 1 from both sides of this new equation gives:

$$xy+\frac{x}{z}+\frac{1}{yz}=1$$

Multiply the above equation by *z* to get:

$$xyz+(x+\frac{1}{y})=z$$

We are given that $x+\frac{1}{y}=2$, so *xyz* + 2 = *z*. Subtracting 2 from both sides of the equation gives *xyz* = *z* − 2, whose value changes with *z*. Thus, *xyz* does **not** have just one possible value.

8. We can rewrite $y+\frac{1}{z}=1$ as $y=1-\frac{1}{z}$ by subtracting $\frac{1}{z}$ from both sides of the equation. So:

$$x+\frac{1}{y}=x+\frac{1}{1-\frac{1}{z}}=x+\frac{z}{z-1}=1$$

Subtracting *x* from both sides of the equation, then multiplying both sides by *z* − 1 gives:

$$\frac{z}{z-1}=1-x$$

$$z=(1-x)(z-1)$$

$$z=z-1-xz+x$$

Subtract *z* and *x* from both sides of the equation, then add 1 to both sides to get *xz* = *x* − 1. Finally, divide both sides of the equation by *x*:

$$z=\frac{x-1}{x}=1-\frac{1}{x}$$

$$z+\frac{1}{x}=1$$

Therefore, **yes**, $z+\frac{1}{x}$ must also equal 1.