Warm-Up!

Try these problems before watching the lesson.

1. Solve each of the following for y:
   (a) If $y = 2x - 7$, and $x = 3$.
   (b) If $y = \frac{2(x-3)}{x+1}$, and $x = 7$.
   (c) If $y = 2x - 3$, and $x = y + 2$.
2. Expand each of the following:
   (a) $(x+1)^2$
   (b) $(t+3)^2$
   (c) $(y-1)^2$
3. State whether each of the following is the square of a binomial:
   (a) $c^2 + 2c + 1$
   (b) $u^2 - 4u + 3$
   (c) $a^2 + 6a + 9$
4. Linda has three less than twice as many porcelain dolls as Sarah. Between them, Linda and Sarah have a total of 60 dolls. How many porcelain dolls does Sarah have?
5. Let $x = 0.\bar{7}$, then $10x = 7.\bar{7}$. Use this to show that $0.\bar{7} = \frac{7}{9}$.
6. Find all values of x such that $\sqrt{5-x} - 5 = 9$.
7. Solve for a in the following equation:

$$\left(\frac{2}{3}a - 5\right)\left(5 - \frac{2}{3}a\right) = -81.$$

The Problem

First Problem: If a and b are integers with $a > b$, what is the smallest possible positive value of $\dfrac{a+b}{a-b} + \dfrac{a-b}{a+b}$?

Second Problem: What is the value of x for which $\sqrt{x + \sqrt{x + \sqrt{x + \cdots}}} = 5$?

Follow-up Problems

8. Evaluate $\sqrt{90 + \sqrt{90 + \sqrt{90 + \cdots}}}$ without a calculator. Then, test your answer with a calculator.
9. Find x if $\sqrt{x - \sqrt{x - \sqrt{x - \cdots}}} = 4$.
10. Express $0.00\bar{6}$ as a fraction in simplest form.
11. In the sequence of integers a, b, c, d, e, f, g, each number is equal to the sum of the two numbers that precede it. If $a = 3$ and $f = 49$, what is the value of g in the sequence?
12. What number equals

$$1 + \cfrac{6}{1 + \cfrac{6}{1 + \cfrac{6}{1 + \cdots}}}?$$

13. Show that if a and b are positive, then $\frac{a+b}{2} \geq \sqrt{ab}$. This relationship is called the **Arithmetic Mean–Geometric Mean Inequality**, or **AM–GM** for short.

Share Your Thoughts

Have some thoughts about the video? Want to discuss the problems on the Activity Sheet? Visit the MATHCOUNTS Facebook page or the Art of Problem Solving Online Community (www.artofproblemsolving.com).