

MATHCOUNTS[®]

Difference of Squares



Warm-Up!

Coach instructions:
Give students 5 to 10 minutes to go through the warm-up problems.

Try these problems before watching the lesson.

1. What is the value of $5^2 - 4^2$?

Solving without the difference of squares identity: $5^2 - 4^2 = 25 - 16 = 9$. **Solving with the difference of squares identity:** $5^2 - 4^2 = (5 - 4)(5 + 4) = (1)(9) = 9$.

2. What is the value of $12^2 - 8^2$?

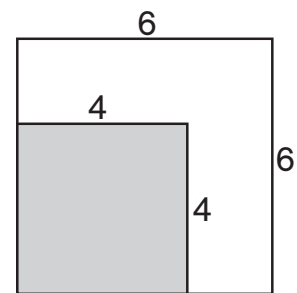
Solving without the difference of squares identity: $12^2 - 8^2 = 144 - 64 = 80$. **Solving with the difference of squares identity:** $12^2 - 8^2 = (12 - 8)(12 + 8) = (4)(20) = 80$.

3. What is the value of $23^2 - 13^2$?

Solving without the difference of squares identity: $23^2 - 13^2 = 529 - 169 = 360$. **Solving with the difference of squares identity:** $23^2 - 13^2 = (23 - 13)(23 + 13) = (10)(36) = 360$.

4. If a square of side length 4 units is placed on top of a square of side length 6 units, what is the area of the non-overlapping region?

This is geometrically the same as asking for the value of $6^2 - 4^2$.
Solving without the difference of squares identity: $6^2 - 4^2 = 36 - 16 = 20$. **Solving with the difference of squares identity:** $6^2 - 4^2 = (6 - 4)(6 + 4) = (2)(10) = 20$.



The Problems

Take a look at the following problems and follow along as they are explained in the video.

5. What is the value of $4^2 - 3^2$?

Solution in video. Answer: 7.

6. What is the value of $212^2 - 211^2$?

Solution in video. Answer: 423.

Coach instructions: After students try the warm-up problems, play the video and have students follow along as the difference of two squares identity is derived. This is a formula students should memorize. You may suggest they take notes during the video.



Piece It Together

Use the skills you practiced in the warm-up and strategies from the video to solve the following problems.

Coach instructions: After watching the video, give students 15 to 20 minutes to try the following problems. Mathletes do not have to use difference of squares to solve these problems, but they should be encouraged to use the identity so they become familiar with it and learn to recognize its structure in problems.

7. What is the value of $2115^2 - 2114^2$?

We can rewrite this expression using the identity and solve:

$$2115^2 - 2114^2 = (2115 - 2114)(2115 + 2114) = (1)(4229) = \mathbf{4229}.$$

8. If $x \textcircled{R} y$ is defined as $x^2 - y^2$, what is the value of $65 \textcircled{R} (8 \textcircled{R} 3)$?

Applying the defined operations to the given expression, we can rewrite and solve:

$$65 \textcircled{R} (8 \textcircled{R} 3) = 65^2 - (8^2 - 3^2)^2 = 65^2 - ((8 - 3)(8 + 3))^2 = 65^2 - ((5)(11))^2 = 65^2 - 55^2 = (65 - 55)(65 + 55) = (10)(120) = \mathbf{1200}.$$

9. What is the value of $(12^2 - 11^2)^2$?

We can re-write this expression using the identity and solve:

$$(12^2 - 11^2)^2 = ((12 - 11)(12 + 11))^2 = ((1)(23))^2 = 23^2 = \mathbf{529}.$$

10. What is the value of $4^4 - 3^4$?

Using properties of exponents we can rewrite this expression and apply the identity:

$$4^4 - 3^4 = (4^2)^2 - (3^2)^2 = (4^2 - 3^2)(4^2 + 3^2) = (4 - 3)(4 + 3)(4^2 + 3^2) = (1)(7)(16 + 9) = (7)(25) = \mathbf{175}.$$

11. What is the value of $25^2 - (25 - 5)(25 + 5)$?

In this problem, instead of factoring a difference of squares into two monomial terms, we

need to take the two monomials and convert them to a difference of squares. We can re-write the expression $25^2 - (25 - 5)(25 + 5) = 25^2 - (25^2 - 5^2) = 25^2 - 25^2 + 5^2 = 5^2 = \mathbf{25}$.

12. What is the value of the expression $\frac{20^2 - 1}{19}$?

We recognize the numerator of the expression as a difference of squares and rewrite the expression: $(20^2 - 1)/19 = (20 - 1)(20 + 1)/19 = (19)(21)/19 = \mathbf{21}$.



Optional Extension

Coach instructions: There are two different provided extensions. If you have extra time, allow Mathletes to choose one or both to work on. You may want to allow them to work in pairs/groups

To extend your understanding and have a little fun with math, try the following activities.

Option 1

See if you can apply the difference of squares formula to find a quicker solution (*hint: you don't have to solve for the unknown*) to the following algebra problems:

If $2x + 3 = 1000$, what is the value of $4x^2 - 9$?

$$4x^2 - 9 = (2x - 3)(2x + 3) = (2x + 3 - 6)(2x + 3) = (1000 - 6)(1000) = (994)(1000) = \mathbf{994,000}$$

If $a - 2 = 1$, then what is the value of $a^4 - 4a^2$?

$$a^4 - 4a^2 = a^2(a^2 - 4) = a^2(a - 2)(a + 2) = (1 + 2)^2(1)(5) = (9)(5) = \mathbf{45}$$

If $x - y = 2$ and $x^2 - y^2 = (55)(59) - (53)(57)$, what is the value of $x + y$?

$$\begin{aligned} (55)(59) - (53)(57) &= (57 - 2)(57 + 2) - (55 - 2)(55 + 2) = 57^2 - 2^2 - (55^2 - 2^2) = \\ 57^2 - 2^2 - 55^2 + 2^2 &= 57^2 - 55^2 = (57 - 55)(57 + 55) = (2)(112) \rightarrow x^2 - y^2 = (2)(112) \rightarrow \\ (x - y)(x + y) &= (2)(112) \rightarrow (2)(x + y) = (2)(112) \rightarrow x + y = \mathbf{112} \end{aligned}$$

Option 2

Similar to difference of squares, there is an identity formula for a difference of cubes.

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Using a geometric representation of difference of cubes, similar to the approach used to derive the difference of squares formula in the video, prove the difference of cubes formula to be true.

If we take a cube of side length a and place a cube of side length b inside it, with $a > b$, then we can divide the space outside of the smaller cube into three rectangular prisms (see figures below). The prisms have volumes $(a - b)(a)(a)$, $(a - b)(a)(b)$ and $(a - b)(b)(b)$. Adding the three volumes together and factoring out $(a - b)$ give us out identity $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

