MATHCOUNTS® Problem of the Week Archive

It's a New Year! – January 1, 2024

Problems & Solutions

Now that 2024 is finally here, let's see what we can do with this new number!

If the perimeter of a particular square is 2024 inches, what is the area of the square, in square inches? Express your answer as a decimal to the nearest hundredth.

Each of the four sides of a square is the same length. So, if one side is s inches, then the perimeter is 4s inches. If 4s = 2024, dividing both sides by 4 shows that s = 506 inches. The area of the square is $A = s^2$, so $A = (506)^2$ = ***256,036*** *square inches.*

What is the median of the first 2024 positive integers? Express your answer as a decimal to the nearest tenth.

Since we are considering the first 2024 positive integers, we can divide this in half and see that there are the first 1012 and then the second 1012, with no integer right in the middle. The median is then the average of the two middle-most terms, which would be the 1012^{th} and 1013^{th} terms, or 1012 and 1013. The average of these two values is ***1012.5****.*

In the arithmetic sequence –7, –4, –1, …, what is the first term greater than 2024?

We can see that the terms are increasing by three each time. If we continue the pattern, we see the sequence is –7, –4, –1, 2, 5, 8, … and each term is one less than a multiple of 3. (Notice that starting with 2, we can consider that we need to add 3x to get as close to 2024 as we can, which will be one less than a multiple of 3.) Knowing our divisibility rules for 3, we know that 2001 is divisible by 3, and therefore, 2000 is a term in the sequence. This means that if we continue the sequence from here, we'll get 2000, 2003, 2006, 2009, 2012, 2015, 2018, 2021, 2024 and 2027. So, the first term in the sequence greater than 2024 is ***2027****.*

If it is noon right now, what time will it be in 2024 hours? Be sure to indicate a.m. or p.m.

We can divide 2024 by 24 and see that there are 84.33333… full days in 2024 hours. This tells us that in 84 full days, it will again be noon. Then, we will have 0.33333… of a day left. Multiplying, we can see that 84 days used up 84 × 24 = 2016 of the 2024 hours, so there are 2024 – 2016 = 8 hours left. So, this puts our time at ***8:00 p.m****.*

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