

MATHCOUNTS® Problem of the Week Archive

Getting Ready for School – August 21, 2023

Problems & Solutions

The date of the beginning of the school year has long been a topic of debate in many areas across the United States, but the trend is leaning towards starting school earlier rather than later. Basically, in the last 10 years, the percent of public schools going back to school before Labor Day has increased by 63%. It is now estimated that 75% of public schools are back in session before Labor Day, rather than after. What percent of public schools were back in session before Labor Day ten years ago, to the nearest whole percent?

*We are looking for the initial percentage before a 63% increase raised it to 75%. If we increase a number by 63%, we are essentially multiplying it by 1.63. So, we have an unknown “x”, which is multiplied by 1.63 and becomes .75. Our equation looks like $1.63x = .75$. Dividing both sides by 1.63 shows us that $x = .4601227$ or **46%**, to the nearest whole percent.*

School systems around the country are dealing with major teacher shortages and have not been able to find a remedy for the problem. Suppose Lincoln Middle School had 483 students and averaged 23 students per teacher during the 2022-2023 school year. If Lincoln Middle School’s student population increases by 14% this year and 10% of last year’s teachers leave, how many new teachers will Lincoln need to hire to keep its average of 23 students per teacher? Express your answer to the nearest whole number.

First, let’s find out how many teachers Lincoln Middle School had during the 2022-2023 school year. Dividing 483 by 23, we see that there were 21 teachers last year. This year, there is an increase of 14% for its student population, which means there are $483(1.14) = 550.62$ or 551 students. We want to keep the same student-to-teacher ratio, so dividing 551 by 23, we see that we will need 23.956522 or 24 teachers for this school year. However, we need to remember that we lost 10% of the teachers from last year, so we only have 90% of them returning, which is $.90(21) = 18.9$ or 19 teachers. Therefore, Lincoln Middle School will need $24 - 19 = 5$ new teachers for the 2023-2024 school year.

It’s the first day of school and you are excited that you have the same 1st period class (Introduction to Theater) as your best friend. You both walk into the classroom and see that you are the first two students to arrive and that there are 20 chairs arranged in a circle (numbered 1-20, in order, with chair #20 next to chair #1). The teacher announces that there are exactly 20 students in the class, and she will be randomly assigning a seat to each student. To do this, she will start with the first student to arrive to class and pull a number (1-20) from a hat to determine which seat the student should sit in. Then she will do the same for the second student to arrive, the third student, and so on until each student has been assigned to a seat. What is the probability that you will be sitting next to your best friend? Express your answer as a common fraction.

Since you and your friend were the first two students to arrive to the classroom, we know that all of the seats are empty as the two of you are getting your seat assignments. Since you are the first to be seated, you could be placed in any of the chairs. So, there are 20 different seats you could get. The first scenario would be that you are placed in seat #1 (which is a $1/20$ chance). Then, your friend would need to get placed in seat #2 or seat #20 (which is a $2/19$ chance). Therefore, that scenario has a $(1/20)(2/19) = 1/190$ chance of happening. The second scenario would be that you get placed in seat #2 (which is a $1/20$ chance). Then, your friend would need to get placed in seat #3 or seat #1 (which is a $2/19$ chance). Therefore, this second scenario has a $(1/20)(2/19) = 1/190$ chance of happening. Notice that there are 18 more scenarios (since there are 18 more seats that you could be assigned to), and each scenario has a $1/190$ chance of happening. Therefore, the probability of you and your friend being seated next to each other is $20(1/190) = \mathbf{2/19}$, as a common fraction.

Another way of looking at it: Wherever you are seated, there will be 19 seats left, two of which will be next to you. So, your placement really doesn't matter, and your friend will have a $\mathbf{2/19}$ chance of being placed next to you.

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