

# MATHCOUNTS® Problem of the Week Archive

## End of Summer Pool Party – August 14, 2023

### Problems & Solutions

A town in South Dakota decides to have an “End of the Summer” pool party and invites all of the households in the town that have school aged children.

Twenty-seven households are invited to the party and the ratio of 5-person households to 4-person households to 3-person households to 2-person households invited is 1:4:3:1, respectively. (The invite list did not include households of any other sizes.) How many people from households with less than 4 people are invited to the party?

*To determine how many households of each size are invited, first set up a proportion for each of the household sizes that are smaller than 4. For two-person households, we have  $1/9 = x/27$ , and for three-person households, we have  $3/9 = y/27$ . Now cross multiply and divide each proportion to get  $9x = 27 \rightarrow x = 3$  two-person households are invited, and  $9y = 81 \rightarrow y = 9$  three-person households were invited. Now we'll multiply the number of two- and three-person households each by the number of people in that size household, and add the results:  $(3)(2) + (9)(3) = 6 + 27 = 33$  people from households with less than four people are invited.*

On the day of the pool party, 75% of the invited people attend. If 1 out of every 12 of the attendees brings one uninvited guest each, how many people attend the pool party?

*Set up proportions, cross multiply and divide to determine the number of households of each size that were invited. We have  $4/9 = a/27 \rightarrow 9a = 108 \rightarrow a = 12$  four-person households were invited, and  $1/9 = b/27 \rightarrow 9b = 27 \rightarrow b = 3$  five-person households were invited. From the last question, we know that 3 two-person households were invited and 6 three-person households were invited. Now multiply the number of households of each size by the number of people in that size household and add the results to get  $(3)(2) + (9)(3) + (12)(4) + (3)(5) = 6 + 27 + 48 + 15 = 96$  people were invited. Multiply the number of people invited by the decimal form of the percentage of invitees that attended and we see that  $(96)(0.75) = 72$  of the invited people attend. Setting up a proportion to determine how many of the attendees brought an uninvited guest, we get  $1/12 = c/72$ . When we cross-multiply and divide, we find that  $12c = 72 \rightarrow c = 6$  people each bring 1 uninvited person, so a total of 6 uninvited guests are in attendance. Add the number of uninvited guests to the number of invited guests and we find the total number of people in attendance at the pool party was  $72 + 6 = 78$  people.*

The party planning committee planned for only 70 people to attend, and it purchased enough supplies to feed each attendee two  $\frac{1}{4}$ -lb hamburgers. Because they underestimated the number of attendees, how many additional pounds of meat must be purchased so that everyone receives two  $\frac{1}{4}$ -lb hamburgers?

*Multiply the number of hamburgers each guest ate by the weight of each hamburger, in pounds, to get  $(2)(\frac{1}{4}) = \frac{1}{2}$  lb of hamburger per person. There are  $78 - 70 = 8$  more attendees than had been planned for. Multiply the number of pounds of hamburger each guest gets by the number of extra people and we see that  $(\frac{1}{2})(8) = 4$  lbs of additional meat are needed.*

At the beginning of the pool party, the rectangular pool, which is 50 ft by 25 ft with an average depth of 5 ft, was completely filled with water. By the end of the day, the water level had lowered by 6 in. By what percent had the amount of water in the pool decreased over the course of the day?

*Multiply the length, width, and depth of the pool to calculate the volume of water originally in the pool, in cubic feet, to get  $(50)(25)(5) = 6250 \text{ ft}^3$ . Multiply the length, width, and the number of feet the depth decreased by to calculate the volume of water lost throughout the day. We get  $(50)(25)(.5) = 625 \text{ ft}^3$ . Divide the amount of water lost by the amount of water originally in the pool and multiply by 100 to calculate the percentage of water lost and the result is  $(625/6250)(100) = 10\%$  of the water was lost.*

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