MATHCOUNTS® Problem of the Week Archive

Wrapping Up the School Year – June 5, 2023

Problems & Solutions

Final exams often accompany the final days of school. Sherry has had four tests so far in her science class, each worth 1/6 of her final grade. She scored 87%, 89%, 92% and 91% on these tests. Her final exam is worth the rest of her final grade. What percentage does Sherry need to score on the final exam to guarantee a final average of at least 90%? Express your answer to the nearest tenth.

Since Sherry has had 4 tests, each worth 1/6 of her final grade, the final exam is worth 1 – 4/6 = 1 – 2/3 = 1/3 of her final grade. To determine what she needs to score to receive at least a 90%, we can set up the following equation: 1/6 × 87 + 1/6 × 89 + 1/6 × 92 + 1/6 × 91 + 1/3 × e = 90. Solving for e, we get 1/6 × (87 + 89 + 92 + 91) + 1/3 × e = 90 → 87 + 89 + 92 + 91 + 2e = 540 → 359 + 2e = 540 → 2e = 181 → e = ***90.5****.*

Later, Sherry took her Algebra exam. Her teacher added one extra credit question at the end that read: Find two integer values of *x* such that the sum of any two members of the ordered triple {*x*, –4, 29} is a perfect square. What are the two integer values that Sherry can write to earn extra credit?

There are three sums to be aware of: x + (– 4), x + 29 and 29 + (– 4). The latter, 29 + (–4) equals 25, which is already a perfect square. So, let's focus on the other two. If we want both x – 4 and x + 29 to be perfect squares, then we should look for two perfect squares that have a difference of 33 because x + 29 – (x – 4) = 33. If we start writing out our perfect squares and looking at the differences, we will notice that 49 – 16 = 33. So, one value of x will have to satisfy x – 4 = 16 and x + 29 = 49. Solving for x we get 20. But the question asked for two values. If you look at differences between each perfect square starting at $1^2 = 1$ and $2^2 = 4$ and continuing, you will notice that the smallest difference is 4 – 1 = 3 and the differences increase by 2 each time: 9 – 4 = 5, 16 – 9 = 7, 25 – 16 = 9, etc. We are looking for a difference of 33. This will occur at $17^2 - 16^2 = 289 - 256 = 33$. The other x value will therefore satisfy the equations x – 4 = 256 and x + 29 = 289. Solving for x, we get x = 260. So, the two values Sherry can write are ***20*** *and* ***260****.*

After finishing up her exams, Sherry needed to turn in the padlock for her locker. Unfortunately, Sherry rarely used her locker during the year and had forgotten her three-number combination. All she could remember was that it formed an arithmetic sequence, the first number was 6, and the numbers increased in value. If the largest number on the padlock is 45, how many different combinations are there that satisfy these conditions?

The smallest value the number could increase by in the arithmetic sequence on the padlock is 1, giving a combination of 6, 7, 8. What we need to know now is the largest possible increase between numbers in the sequence. Since the largest number possible is 45, we should consider if a sequence 6, x, 45 is possible. It is not, because the difference between 45 and 6 is 39, which would mean the increase in the three-term sequence would be a decimal number. Let's look at the next largest number, 44. In the sequence 6, x, 44, the increase between terms is (44 – 6) ÷ 2 = 38 ÷ 2 = 19. This means we can have any increase between 1 and 19, and therefore, there are ***19*** *possible combinations.*

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