

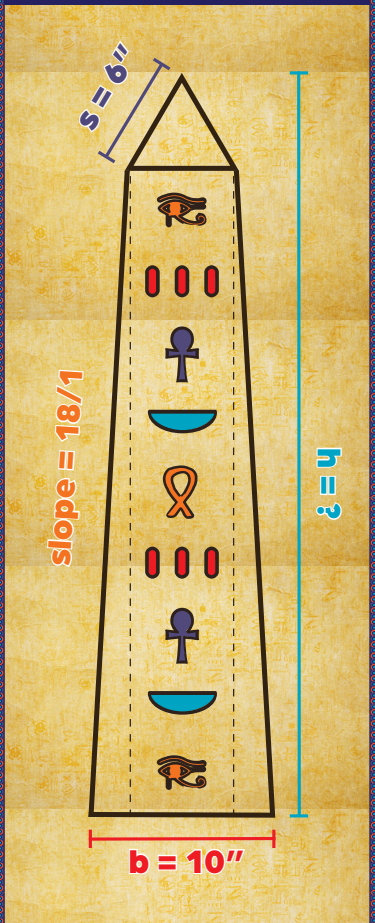
MATHCOUNTS 2021–2022 Fall Newsletter Poster Solution

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This obelisk is drawn with an isosceles trapezoid topped by an equilateral triangle of **side length 6"**. If the trapezoid has a **base of 10"** and a leg of **slope $18/1$** , how tall is the obelisk?



Express your answer in simplest radical form.



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The height of the obelisk is the combined heights of the equilateral triangle and the isosceles trapezoid.

The equilateral triangle has side length 6". The altitude drawn from the apex of the obelisk to the base of the equilateral triangle creates two congruent 30-60-90 right triangles, as shown in Figure 1, each with a short leg

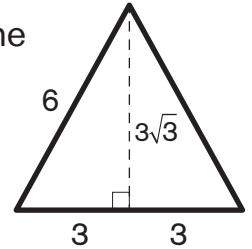


Figure 1

of length 3" and hypotenuse of length 6". By properties of 30-60-90 right triangles, the altitude of the equilateral triangle, which is also the long leg of both right triangles has length $3\sqrt{3}$ ". Now, to determine the height of the trapezoid, we will use the fact that its leg has slope $18/1$.

Suppose the bottom left vertex of the obelisk is on a coordinate plane at (0, 0) as shown in Figure 2. Since

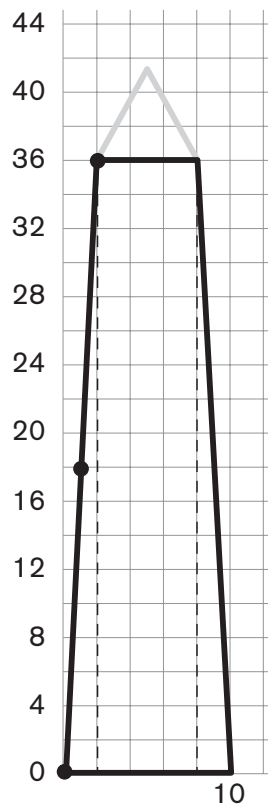


Figure 2

our obelisk is symmetrical and has a base of 10" and the equilateral triangle has a base of 6", we know that the top left vertex of the trapezoid is at some point (2, y). From (0, 0), moving 1" horizontally and 18" vertically takes us to (1, 18). From there, moving another 1" horizontally and 18" vertically takes us to (2, 36), which is the top left vertex of the trapezoid. So, the trapezoid has height 36". Therefore, the obelisk is $3\sqrt{3} + 36 \approx 41.2$ " tall.