MATHCOUNTS®

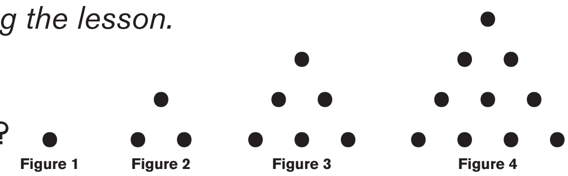
Representing Patterns Numerically

Warm-Up!

Coach instructions: *Give students around 10 minutes to go through the warm-up problems.*

Try these problems before watching the lesson.

1. A sequence of figures is created with dots as shown. If this pattern continues, how many dots will be in Figure 5?



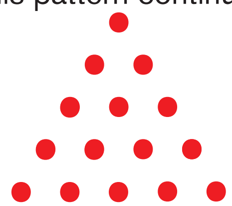
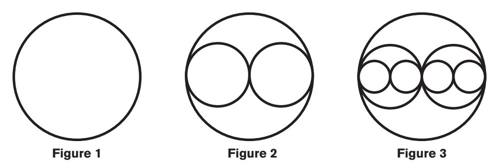


Figure 5

The pattern is a series of dots in a triangular formation. Each figure, one row is added with one more dot than the previous row. Students may have chosen to draw out figure 5, as shown. The total number of dots is 5 + 4 + 3 + 2 + 1 = 15 dots.

2. A sequence of figures is created by inscribing circles in the pattern shown. How many circles will there be in Figure 4?



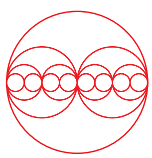
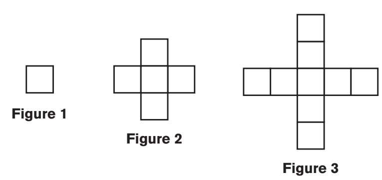


Figure 4

Students may have followed the pattern of inscribing two circles in each of the smallest circles and drawn out figure 4, as shown. Figure 4 has a total of 1 + 2 + 4 + 8 = 15 circles.

3. A sequence of figures is created with squares as shown. If the pattern continues, how many squares will be in Figure 4?



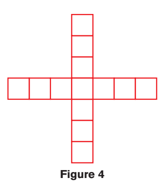


Figure 4

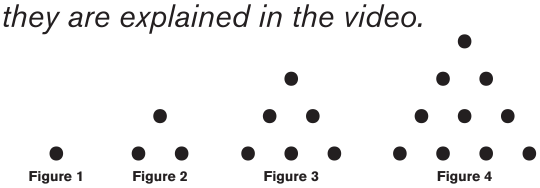
Following the pattern, each sequential figure a square is added to the left, right, top and bottom of the previous figure. So each figure has four more squares than the last. Figure 4 will have 1 + 4 + 4 + 4 = 13 squares.

The Problems

Coach instructions: *After students try the warm-up problems, play the video and have them follow along with the solutions.*

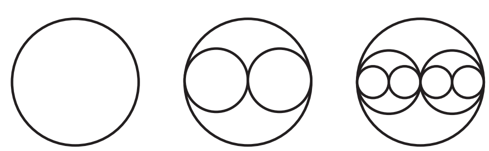
Take a look at the following problems and follow along as they are explained in the video.

4. A sequence of figures is created with dots as shown. If this pattern continues, how many dots will be in Figure 10?



Solution in video. Answer: 55 dots.

5. A sequence of figures is created by inscribing circles in the pattern shown. How many circles will there be in Figure 7?



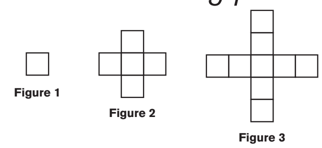
Solution in video. Answer: 127 circles

Piece It Together

Coach instructions: *After watching the video, give students 15 minutes to try the next four problems.*

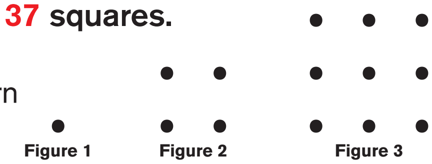
Use the skills you practiced in the warm-up and strategies from the video to solve the following problems.

6. A sequence of figures is created with squares as shown. If the pattern continues, how many squares will be in Figure 10?



This pattern starts with one square and add 4 squares in each subsequent figure. Figure 2 has 1 + 4 = 5 squares. Figure 3 has 1 + 4 + 4 = 1 + 4(2) = 9 squares. And, as we solved in the warm-up, figure 4 has 1 + 4 + 4 + 4 = 13 squares. This means that figure 10 will have 1 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 = 1 + 4(9) = 1 + 36 = 37 squares.

7. A sequence of figures is created with dots as shown. If this pattern continues, how many dots will be in Figure 10?



In this pattern, Figure 1 has one dot, Figure 2 has four dots and Figure 3 has nine dots. Notice that the number of dots in each is a series of squares, $1^2 = 1$, $2^2 = 4$ and $3^2 = 9$. This means that figure 10 will have $10^2 = 100$ dots.

8. A sequence of figures is created by lines and dots as shown. If each segment from a dot to its nearest neighbors has length 1, then how many triangles of perimeter 3 will there be in Figure 7?

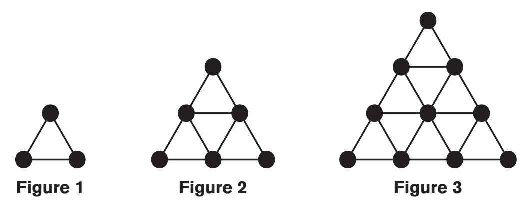
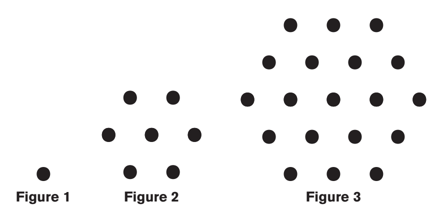


Figure 1 has one triangle, Figure 2 has four and Figure 3 has nine. Each subesquent figure we add the next consecutive odd integer. Figure 7 will therefore have 1 + 3 + 5 + 7 + 9 + 11 + 13 = 49 triangles with perimeter three units.

9. A sequence of figures is created with dots as shown. If this pattern continues, how many dots will be in Figure 5?

In this pattern, the dots are forming a hexagon shape. Each subsequent figure constructs another hexagon around the previous figure. The numeric pattern that emerges is addition of multiples of 6 (note: a hexagon has six-sides). Figure 5 will therefore have 1 + 6(1) + 6(2) + 6(3) + 6(4) = 1 + 6 + 12 + 18 + 24 = 61 dots.



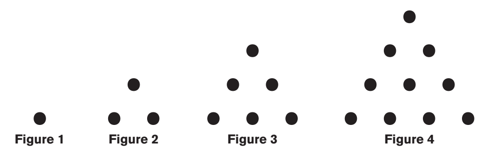
Optional Extension

Coach instructions: *Once your students have completed the problems and feel they have a comfortable understanding of how to represent patterns numerically, let them try to write formulas to define the pattern for any Figure n.*

To extend your understanding and have a little fun with math, try the following activities.

Now, try to write a formula for the each of the patterns we looked at in the Warm-Up and Piece It Together problems. Use the variable n (or other variable of your choosing) where n represents the figure number. This formula should allow you to find the number of items you are counting (dots, circles, squares or triangles) in any figure number by plugging in for n and following the rules of the formula.

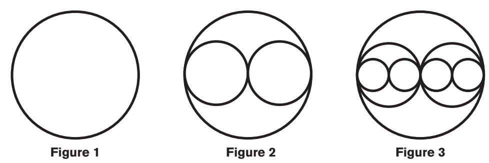
There are 6 different patterns that we looked at in this practice plan. Formulas that can be written for each are as follows:



1. This pattern is a summation of a series of positive integers from 1 to n. This can be represented by the following formula:

$$1 + 2 + 3 + \ldots + (n - 1) + n$$

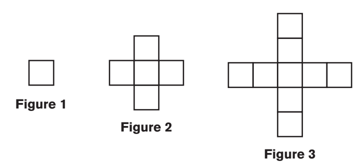
This arithmetic series can also be calculated using the formula $n(n + 1)/2$. Student may want to become familiar with this formula as it often can be used in MATHCOUNTS problems.



2. This pattern is a geometric series of powers of 2 starting with 2^0 and going through 2^n. This can be represented by the following formula:

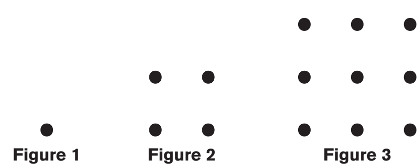
$$2^0 + 2^1 + 2^2 + \ldots + 2^n$$

This sum of this geometric series can also calculated by the formula $2^{n+1} - 1$.



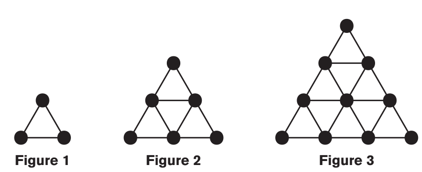
3. In this pattern of squares, each figure adds four additional squares. Figure 2 has four more squares than Figure 1 and Figure 3 has eight more squares than Figure 1. Using this, the following formula for the total number of squares in Figure n can be written:

$$1 + 4(n - 1)$$



4. In this pattern, the number of dots is equal to the square of the figure number, this can be written simply as:

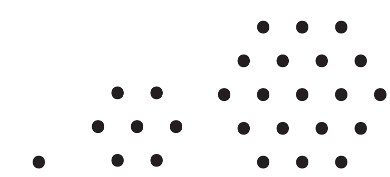
$$n^2$$



5. In this pattern, the number of triangles (with perimeter 3 units) is found by an arithmetic series of odd integers. This formula can be written as follows:

$$1 + 3 + 5 + \ldots + (2n - 1)$$

This arithemetic series of odd integers is equivalent to n^2 — another formula students may want to familiarize themselves with.



6. The hexagon dot pattern, can be counted by adding up multiples of six. The formula can be written as follows:

$$1 + 6(1) + 6(2) + \ldots + 6(n - 1)$$