MATHCOUNTS Minis November 2009 Activity Solutions

Warm-Up!

1. To get from the 1st term of this sequence to the 5th term, we must add the common difference to the 1st term 4 times. Thus, the 5th term of the sequence is 8 + 3(4) = **20**. To get from the 1st term to the 35th term, we must add the common difference to the 1st term 34 times, resulting in 8 + 3(34) = 8 + 102 = **110**.

2. We can determine that the common difference of the arithmetic sequence is 17 – 14 = 3. In order to go from the 1st term to the 21st term we must add the common difference to the 1st term 20 times. The result is 14 + 3(20) = **74**.

3. The nth term of an arithmetic sequence where a is the 1st term of the sequence and d is the common difference can be represented as $\boldsymbol{a + d(n - 1)}$.

The Problem is solved during the November 09 MATHCOUNTS Mini.

Follow-up Problems

4. For sequence A, we know that we must add the common difference of 9 to A's 1st term of 30 a total of 50 times. Sequence A's 51st term is 30 + 9(50) = 480. For sequence B, we must add the common difference of 4 to B's 1st term of 30 a total of 50 times. Sequence B's 51st term is 30 + 4(50) = 230. The difference between these two terms is 480 – 230 = **250**. Notice that since the 1st terms were the same and we were comparing each of their 51st terms, we could have found the ultimate difference by seeing that the common differences differed by 9 – 4 = 5, and that was going to occur 50 times, resulting in an ultimate difference of 5(50) = 250.

5. The common difference in the first sequence is 6, and the common difference in the second sequence is 8. Carrying out the first sequence to the 4th term, we see that it is 20, and we have found a common term. Because the least common multiple of the 6 and 8 is 24, the sequences will have common terms of 20, 20 + 24(1), 20 + 24(2), 20 + 24(3), and so on. Multiplying 24 by 20 gets us to 480, and then adding the 1st term of 20 tells us the term 20 + 480 = 500 is in both sequences. We need the largest common term less than 500. Subtracting 24, we find that it is 500 – 24 = **476**.

6. Going from the 7th term to the 12th term of an arithmetic sequence requires adding the common difference to the 7th term a total of 12 – 7 = 5 times. Therefore we know $48 = 24 + d(5) \rightarrow 24 = 5d \rightarrow d = 24/5$. To get to the 57th term from the 7th term, the common difference of 24/5 must be added 57 – 7 = 50 times, resulting in 24 + (24/5)(50) = 24 + 240 = **264**.

Further Exploration

7. The first six terms are **2/9**, (2/9)(3) = **2/3**, (2/3)(3) = **2**, 2(3) = **6**, 6(3) = **18**, 18(3) = **54**.

8. To get from the 1st term to the 6th term, the first term (–8) must be multiplied by the common ratio (–1/2) a total of 6 – 1 = 5 times. The 6th term would be $(-8) \times (-1/2)^5$ = **1/4**.

9. The *n*th term of a geometric sequence can be expressed as $\boldsymbol{a \times r^{(n-1)}}$ where *a* is the 1st term of the sequence and *r* is the common ratio.

10. To get from the 32nd term of a geomteric sequence to the 41st term, the 32nd term must be multiplied by the common ratio a total of 41 – 32 = 9 times. Thus, we have $36 = 24 \times r^9$. Dividing both sides of the equation by 24 and then simplifying, we have $3/2 = r^9$. Notice that going from the 41st term to the 59th term, we must multiply the 41st term by the common ratio a total of 59 – 41 = 18 times. If $r^9 = 3/2$, then $r^{18} = r^9 \times r^9 = (3/2)(3/2) = 9/4$. To find the 59th term, we must simplify 36 × (9/4), which is **81**.