



An arithmetic sequence is a sequence of numbers in which the difference between each term and the next is always the same. Here are some examples:

1, 2, 3, 4, 5, 6, 7

6, 11, 16, 21, 26

99, 88, 77, 66, 55

We can use “...” to say that a sequence continues indefinitely, so “1, 2, 3, 4, 5, 6, ...” refers to the arithmetic sequence of all positive integers. We say that the constant difference between two consecutive terms is the “common difference.” So, our three example sequences above have common differences, 1, 5, and -11 , respectively. Try the following problems before watching the lesson.

1. Suppose an arithmetic sequence has first term 8 and common difference 3. What is the 5th term of the sequence? What is the 35th term of the sequence?
2. Suppose the first three terms of an arithmetic sequence are 14, 17, 20. What is the 21st term of the sequence?
3. Suppose the first term of an arithmetic sequence is a and the common difference is d . Find an expression for the n^{th} term in terms of a , d , and n . Test your expression by making sure it gives the same answers you found for the first two problems.



We have two problems this month!

- Two arithmetic sequences A and B both begin with 30 and have common differences of absolute value 10, with sequence A increasing and sequence B decreasing. What is the absolute value of the difference between the 51st term of sequence A and the 51st term of sequence B ?
- The arithmetic progressions $\{2, 5, 8, 11, \dots\}$ and $\{3, 10, 17, 24, \dots\}$ have some common values. What is the largest value less than 500 that they have in common?



Follow-up Problems

- Two arithmetic sequences A and B both begin with 30. Sequence A has common difference 9 and sequence B has common difference 4. What is the difference between the 51st term of A and the 51st term of B ?
- The arithmetic progressions $\{2, 8, 14, \dots\}$ and $\{4, 12, 20, \dots\}$ have some common values. What is the largest value less than 500 that they have in common?
- If the 7th term of an arithmetic sequence is 24 and the 12th term is 48, then what is the 57th term?




Further Exploration

In a geometric sequence, after the first term, the ratio between each term and the previous term is the same. We call this ratio the “common ratio.” So, for example, the sequence

$$1, 2, 4, 8, 16, 32$$

is a geometric sequence with common ratio 2.

- Find the first 6 terms of a geometric sequence with first term $\frac{2}{9}$ and common ratio 3.
- If the first term of a geometric sequence is -8 and the common ratio of the geometric sequence is $-\frac{1}{2}$, then what is the 6th term of the sequence?
- If the first term of a geometric sequence is a and the common ratio of the sequence is r , then what is the n th term in terms of a , r , and n ?
- If the 32nd term of a geometric sequence is 24 and the 41st term of the sequence is 36, then what is the 59th term?



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