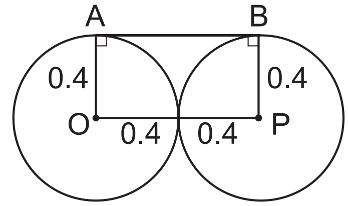
# MATHCOUNTS Minis October 2010 Activity Solutions

**Warm-Up!**

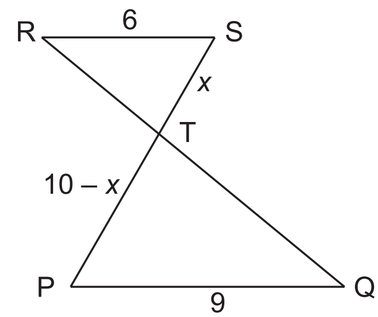
1. The formula for the circumference, C, of a circle is $C = \pi \times d$, where $d$ is the diameter of the circle. The radius of our circle is 12 inches, so the diameter is 24 inches and the circumference is $\pi \times 24 =$ **24π** inches. This is 75.4 inches, to the nearest tenth.

2. If we label the points of tangency as A and B on circles O and P, respectively, we create rectangle ABPO. The shorter sides of this rectangle are each 0.4 inches, or the radius of the circles, and the longer sides, including segment AB, are each **0.8** inches.




3. Determining triangles RTS and QTP are similar enables us to write the following proportion:

$\frac{RT}{QT} = \frac{TS}{TP} = \frac{SR}{PQ}$. Knowing SP = 10, we can let ST = $x$ and then PT = 10 – $x$. Going back to our original proportion and filling in some of the known values, we see $\frac{TS}{TP} = \frac{SR}{PQ} \rightarrow \frac{x}{10 - x} = \frac{6}{9} \rightarrow \frac{x}{10 - x} = \frac{2}{3}$. Using cross products, we have $20 - 2x = 3x \rightarrow 20 = 5x \rightarrow x = 4$. This means ST = **4** units and PT = **6** units.




**The Problem** is solved during the MATHCOUNTS Mini.

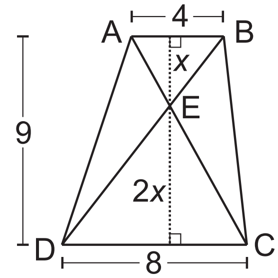
**Follow-up Problems**

4. As described in the Mini, we can see that we have two "tangent segments" that are the straight portions of the belt, and each of these has the length of two radii. Therefore, the two straight portions of the belt total 2(2 × 10) = 40 cm. Also as explained in the Mini, the two curved portions together make a complete circle, and account for 360 degrees. Each curved portion is a half-circle, and together they total the circumference of one circle: 20π cm. The total length of the belt is then **40 + 20π** cm.

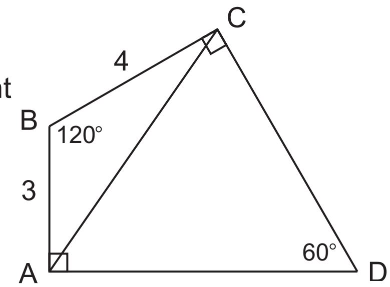
5. Rather than two straight portions of the belt, there are four. These four segments account for 20 cm each, or 80 cm total. Again, the curved portions of the belt (four of them) together complete an entire circle and account for 20π cm. The total length of the belt is **80 + 20π** cm.

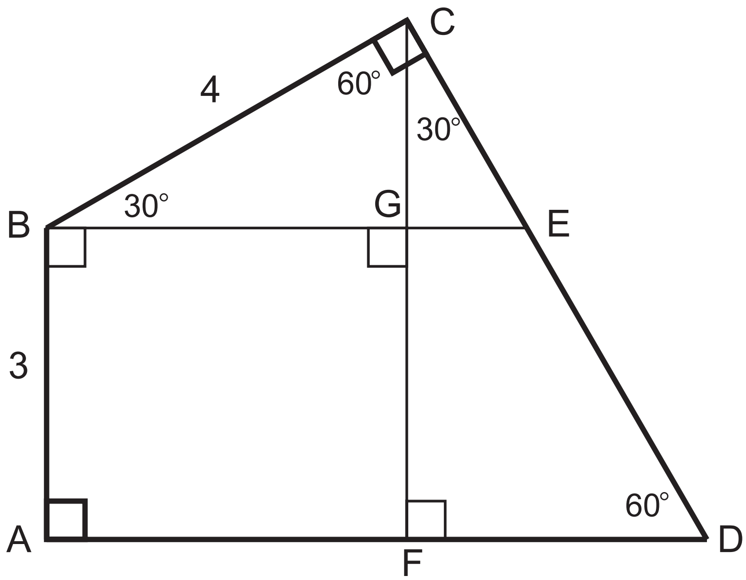
6. The six straight portions of the belt (6 × 20) and the six curved portions of the belt (20π) result in a belt that is **120 + 20π** cm long.

7. From the given information, we can deduce triangles ABE and DCE are similar. Their corresponding bases are in a ratio of 4:8, meaning their scale factor is 1:2. Using segments AB and CD as their bases, the triangles' heights are in a 1:2 ratio and sum to 9 units. Using $x + 2x = 9$, we can calculate the heights to be 3 and 6 units, respectively. The area of triangle ABE is then (1/2)(4)(3) = **6** square units.



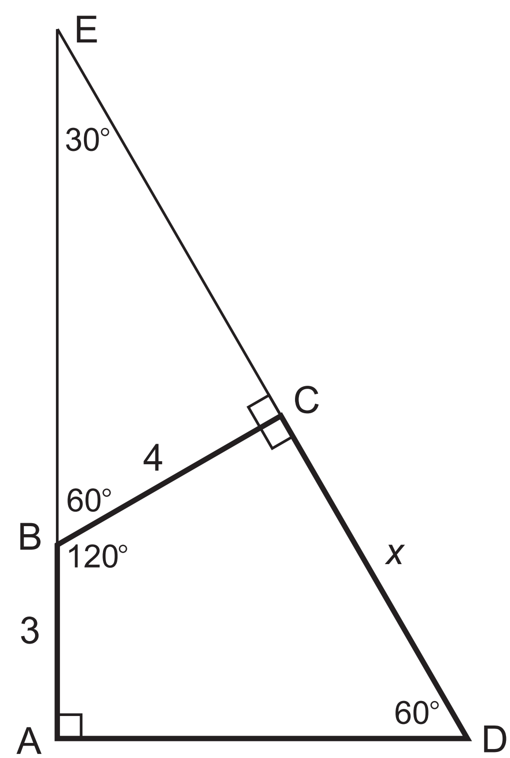
8. One solution involves taking segment AC out of the figure and drawing in the segment from point B that is parallel to side AD and the segment from point C that is perpendicular to side AD. To the right is the original figure, and below is the figure with our new segments. From the parallel lines and the fact that the sum of the angles of a triangle is 180 degrees, we also know all of the degree measures shown.




We now have triangles BCG and CDF that are each 30-60-90 triangles. Using a knowledge of the relationships between the sides of 30-60-90 triangles [hypotenuse = 2 × (short leg); long leg = $\sqrt{3}$ × (short leg)], we can start with triangle BCG and see that if its hypotenuse is 4 units, then CG = 2 units. Seeing that quadrilateral ABGF must be a rectangle, we know AB = GF = 3 units, and then FC = 2 + 3 = 5 units. Segment FC is the long leg of triangle FCD, so FD = $5/\sqrt{3}$ units and CD = $10/\sqrt{3}$ = **$(10\sqrt{3})/3$** units.

A second solution starts by again omitting segment AC and then extending segments AB and DC to intersect at point E. We now can determine all of the angle measures, and we have the figure shown to the right. Using our knowledge of the relationships between the sides of 30-60-90 triangles, we see the short leg of triangle EBC is 4 units, so its hypotenuse (segment EB) measures 8 units, and its long leg (segment CE) measures $4\sqrt{3}$ units. Now we know segment AE is 3 + 8 = 11 units. This is the long leg of 30-60-90 triangle EDA. Its short leg must be $11/\sqrt{3}$, and its hypotenuse (segment ED) is $22/\sqrt{3} = (22\sqrt{3})/3$. Letting CD = $x$, we now have $x$ = ED – CE $\rightarrow$ $x = (22\sqrt{3})/3 - 4\sqrt{3}$ $\rightarrow$ $x = (22\sqrt{3})/3 - (12\sqrt{3})/3$ = **$(10\sqrt{3})/3$** units.




9. "Goodbye to the students, Harvey."