MATHCOUNTS® Minis January 2010 Activity Solutions

Warm-Up!

1. With only yellow socks and blue socks, choosing **3** socks is the smallest number of socks that guarantees getting two socks of the same color. In choosing 3 socks, you could pull out three blue socks (BBB), three yellow socks (YYY), two blue socks and one yellow sock (BBY) or two yellow socks and one blue sock (BYY). In each of these scenarios, there are at least two of one of the colors of socks.

2. With four different colors of socks, the largest number of socks that can be chosen without choosing two socks of the same color is **4**. If you choose five socks, at least two of them must be the same color, since there are only four colors of socks.

3. Even numbers have a remainder of 0 when divided by 2, and odd numbers have a remainder of 1 when divided by 2. There are no other possible remainders when dividing any of the numbers 1 through 16 by 2. So, Jillian could draw a maximum of **2** slips without choosing two numbers that have the same remainder when divided by 2.

4. Between 1 and 16, inclusive, there are 8 even numbers and 8 odd numbers. It would be possible for Jillian to draw all 8 even numbers in a row first (or all 8 odd numbers in a row first), and therefore, for the first 8 draws to all have the same remainder when divided by 2. However, in this scenario, the ninth draw must be a different remainder. So, the maximum number of slips that Jillian can draw without choosing two that have different remainders is **8**.

The Problems are solved in the MATHCOUNTS® Mini video.

Follow-up Problems

5. Using Richard's "Monkey Barrel Principle" (or Pigeonhole Principle), we need to first look at what perfect squares we have between 1 and 30, inclusive. The perfect squares in this range are 1, 4, 9, 16 and 25. So, Jillian could only draw one of these slips without obtaining a product that is a perfect square. Similarly, 2, 8 and 18 are each double a perfect square. By the same reasoning, Jillian could only draw one of these slips without obtaining a product that is a perfect square. The same is true for numbers that are triple a perfect square (3, 12, 27); numbers that are five times a perfect square (5, 20); numbers that are six times a perfect square (6, 24); and numbers that are seven times a perfect square (7, 28). If Jillian selects one slip from each of these "barrels," she will have 6 numbers, none of whose product is a perfect square. Jillian could then draw all 13 of the remaining numbers in this range (10, 11, 13, 14, 15, 17, 19, 21, 22, 23, 26, 29, 30) and still not obtain a product that is a perfect square. Therefore, the maximum number of slips Jillian can draw without obtaining a product that is a perfect square is 6 + 13 = **19**.

6. Per the problem in the video, Jillian could draw a maximum of 11 slips between 1 and 16, inclusive, without obtaining a product that is a perfect square. Following the same process, we find that Jillian could also draw a maximum of 11 slips between −16 and −1, inclusive, without obtaining a product that is a perfect square. (Because 0 is a perfect square, it does not change these outcomes.) Therefore, Jillian could draw a maximum of 11 + 11 = **22** slips without obtaining a product that is a perfect square.

7. There are significantly more than a million people in New York City (in the 2020 census, there were 8,804,190 people to be exact). There are significantly less than a million hairs on a human head (generally, about 100,000 to 200,000). Therefore, it is clear that it would *not* be possible for every person in New York City to have a distinct number of hairs. Many pairs of people must have the same number of hairs on their heads.

8. Because we are dividing by 6, there are 6 possible remainders: 0, 1, 2, 3, 4, 5. In a group of 7 integers, it is possible that the first 6 integers all have different remainders, but the seventh integer's remainder must match one of the first six integers' remainders, as there are only six possible remainders.

9. Let's say each person is given 9 pieces of candy. This accounts for 9 × 5 = 45 pieces of candy, which means that the 46th piece of candy must still be given to someone. So, one person would get 10 pieces of candy.

10. For each person to have a different number of friends, the number of friends must be 0, 1, 2, 3, 4, 5, 6 or 7. If someone has 0 friends, then none of the seven people left can have 7 friends, because there are just 6 people left to be friends with. So, at least two people will have the same number of friends.