

MATHCOUNTS® Problem of the Week Archive

Summer Plans – July 31, 2023

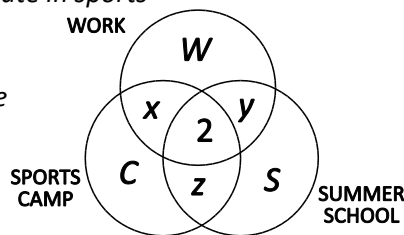
Problems & Solutions

After polling each student in her homeroom about their summer plans, Mrs. Baker discovered that each student planned to participate in one or more of the following activities:

1. Attend summer school
2. Work part-time
3. Attend sports camp

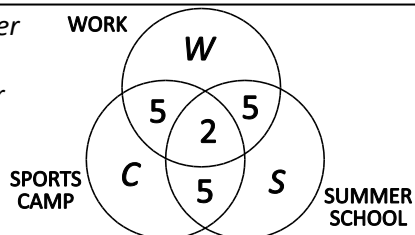
Mrs. Baker learned that 17 of the students in her homeroom plan to participate in at least two of these activities, and exactly two students plan to participate in all three activities this summer. If the number of students participating in exactly two of these activities is equally divided among the three pairs of activities, how many students plan to both attend sports camp and work part-time this summer, but not attend summer school?

We let x , y and z represent the number of students planning to participate in sports camp and work part-time, work part-time and attend summer school, and attend summer school and sports camp, respectively. We are told there are 2 students who plan to participate in all three activities. Since there are 17 students planning to participate in two or more activities, we have $x + y + z + 2 = 17$. We are also told that $x = y = z$. Therefore, $3x + 2 = 17 \rightarrow 3x = 15 \rightarrow x = 5$ students plan to attend sports camp and work part-time this summer.



If $\frac{1}{4}$ of Mrs. Baker's homeroom students who plan to attend summer school do not plan to participate in either of the two other activities, how many students plan only to attend summer school?

Let S represent the number of students who plan only to attend summer school. From the previous problem, we learned that $5 + 5 + 2 = 12$ students plan on participating in summer school and at least one other activity, so $S = \frac{1}{4}(S + 12)$. Solving for S yields $4S = S + 12 \rightarrow 3S = 12 \rightarrow S = 4$. So, **4** students plan only to attend summer school.



According to this poll, 16 of Mrs. Baker's homeroom students plan to participate in only one of these activities. If the number of students who plan only to attend sports camp is three times the number of students who plan only to work part-time, what is the total number of students in Mrs. Baker's homeroom?

We know that 17 students plan to participate in two or more activities, and 16 plan to participate in only one activity. So, there must be $17 + 16 = \mathbf{33}$ students in Mrs. Baker's homeroom.

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